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# A Practical Method for Estimating $L_{eq}$ by Considering Dynamic Range of Measurement Instruments

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**ABTRACT**

The noise evaluation index, $L_{eq}$, plays an important role in the field of noise evaluation and/or regulation problems. At this time, the sound level fluctuation is often measured within a restricted range due to a dynamic range of measurement instruments. This incomplete measurement occurs the lack of measurement accuracy of $L_{eq}$. In this paper, a restoration method of statistical information on the sound level fluctuation is first proposed by using the above incomplete sound level data under the assumption of a standard Gaussian distribution. Next, a practical estimation method $L_{eq}$ is proposed by using the estimated statistical information on the mean value and the variance of the sound level fluctuation. The validity of the proposed estimation method is first confirmed through some simulation experiments. The practical effectiveness of the proposed method is experimentally confirmed by applying it to the actual road traffic noise data.

**Keywords:** Dynamic Range, Sound Level Fluctuation, Environmental Noise, $L_{eq}$ Noise Evaluation Index, Measurement Instrument

## 1. INTRODUCTION

As is well known, the noise evaluation index, $L_{eq}$, plays an important role in the field of noise evaluation and/or regulation problems. In an actual acoustic measurement, the sound level fluctuation is often measured within a restricted range due to a dynamic range of measurement instruments. The $L_{eq}$ noise evaluation index is defined as constant sound level whose sound energy value is equal to an averaged energy of the sound energy of the sound level fluctuation over a total measurement time interval. Therefore, this incomplete measurement occurs the lack of measurement accuracy of $L_{eq}$.

From the above viewpoint, a signal processing method for the restoration of statistical information on the sound level fluctuation is very important to evaluate $L_{eq}$ by using the above incomplete sound level data. On the other hand, in the previous study [1], a practical estimation method has been derived under the assumption of a standard Gaussian distribution with the statistical information on the mean value and the variance of the sound level fluctuation.

In this paper, a practical estimation method of $L_{eq}$ is proposed by considering the measurement mechanism due to a dynamic range. More specifically, an estimation method of the mean and variance of the original sound level fluctuation before passing measurement mechanism due to the dynamic range is first proposed under the assumption of Gaussian distribution of the above original sound level fluctuation. Next, a practical estimation method of $L_{eq}$ is proposed by using the previous study together with the estimated values of the mean and variance.

The validity of the proposed estimation method is first confirmed through some simulation experiments. Moreover, the practical effectiveness of the proposed method is experimentally confirmed by applying it to the actual road traffic noise data.

## 2. THEORETICAL CONSIDERATION

The $L_{eq}$ noise evaluation index is defined as constant sound level whose sound energy value is equal to an averaged energy of the sound energy of the sound level fluctuation over a total measurement time interval, as follows :

$$L_{eq} = 10\log_{10}\frac{\langle E\rangle}{E_0}, \qquad (1)$$

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where $E$ denotes the sound energy fluctuation, and <*> denotes an expectation operation with respect to the random variable *. Moreover, $E_0$ denotes the reference sound energy usually taken as $E_0=10^{-12}$[watt/m$^2$].

The relationship between the sound level $x$ and the sound energy $E$ is expressed as follows :

$$x = 10\log_{10}\frac{E}{E_0}. \tag{2}$$

Moreover, $y$ denotes the sound level fluctuation measured within a restricted range due to the dynamic range $[a,b]$ of a measurement instrument. Therefore, this incomplete measurement occurs the lack of measurement accuracy of $L_{eq}$. As contrasted with this dynamic range, it is necessary for sufficient measurement accuracy to employ a suitably fine sampling interval [2,3].

For simplicity of theoretical procedures, let us introduce Gaussian distribution as the probability density function $P_x(\bullet)$ of the original sound level fluctuation $x$, as follows :

$$P_x(x) = \frac{1}{\sqrt{2\pi}\sigma_x} e^{-\frac{(x-\mu_x)^2}{\sigma_x^2}} \left(\equiv N(x;\mu_x,\sigma_x^2)\right), \tag{3}$$

where $\mu_x$ and $\sigma_x^2$ denote respectively the mean value and the variance. According to the pervious study [1], the estimation formula derived based on Gaussian property is given as follows :

$$L_{eq} = \mu_x + 0.115\sigma_x^2. \tag{4}$$

According to the basic concept in the previous studies [4,5], the probability density function $P_y(y)$ after passing the dynamic range can be derived by using $P_x(\bullet)$ of the original sound level fluctuation $x$, as follows :

$$P_y(y) = \delta(y-a)\int_{-\infty}^{a} P_x(y)dy + P_x(y)u_{a,b}(y) + \delta(y-b)\int_b^{\infty} P_x(y)dy, \tag{5}$$

where $\delta(\bullet)$ denotes Dirac's delta function and $u_{a,b}(y)$ denotes the unit function defined as

$$u_{a,b}(y) = \begin{cases} 1 & (a < y < b), \\ 0 & (y \le a \text{ or } b \le y). \end{cases} \tag{6}$$

Therefore, based on Eq.(5) and an averaging operation with respect to $y$, the values of $\mu_y$ and $\sigma_y^2$ can be respectively given as follows :

$$\begin{aligned} \mu_y &= \int_{-\infty}^{\infty} y P_y(y)dy \\ &= \int_{-\infty}^{\infty} y\Big[\delta(y-a)\int_{-\infty}^{a} P_x(y)dy + P_x(y)u_{a,b}(y) \\ &\quad + \delta(y-b)\int_b^{\infty} P_x(y)dy\Big]dy \\ &= a\Phi\left(\frac{a-\mu_x}{\sigma_x}\right) + \sigma_x^2\left(N(a;\mu_x,\sigma_x^2) - N(b;\mu_x,\sigma_x^2)\right) \\ &\quad + \mu_x\left(\Phi\left(\frac{b-\mu_x}{\sigma_x}\right) - \Phi\left(\frac{a-\mu_x}{\sigma_x}\right)\right) \\ &\quad + b\left(1-\Phi\left(\frac{b-\mu_x}{\sigma_x}\right)\right), \end{aligned} \tag{7}$$

$$\begin{aligned} \sigma_y^2 &= \int_{-\infty}^{\infty} (y-\mu_y)^2 P_y(y)dy \\ &= \int_{-\infty}^{\infty} (y-\mu_y)^2\Big[\delta(y-a)\int_{-\infty}^{a} P_x(y)dy + P_x(y)u_{a,b}(y) \\ &\quad + \delta(y-b)\int_b^{\infty} P_x(y)dy\Big]dy \\ &= (a-\mu_y)^2\Phi\left(\frac{a-\mu_x}{\sigma_x}\right) + \sigma_x^2\left((a-\mu_y)N(a;\mu_x,\sigma_x^2)\right. \\ &\quad \left. -(b-\mu_y)N(b;\mu_x,\sigma_x^2)\right) + \sigma_x^2\left(\Phi\left(\frac{b-\mu_x}{\sigma_x}\right)\right. \\ &\quad \left. -\Phi\left(\frac{a-\mu_x}{\sigma_x}\right)\right) + (b-\mu_y)^2\left(1-\Phi\left(\frac{b-\mu_x}{\sigma_x}\right)\right), \end{aligned} \tag{8}$$

where $\Phi(\bullet)$ denotes the cumulative distribution function of a standard Gaussian distribution defined as

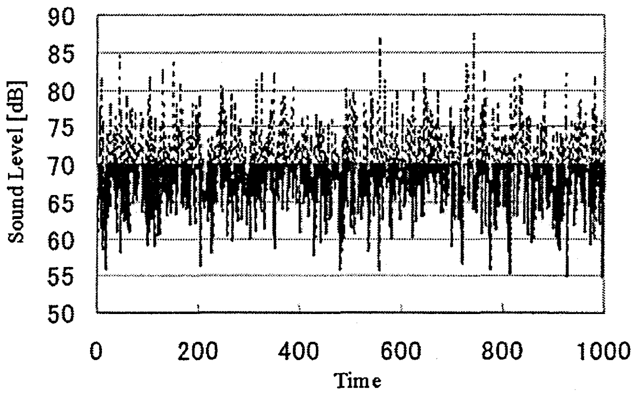
$$\Phi(t) = \int_{-\infty}^{t} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}} du. \tag{9}$$

Therefore, we can obtain Eqs.(7) and (8) as the simultaneous equations by regarding two parameters $\mu_x$ and $\sigma_x^2$ as the unknown variables. By solving these non-linear simultaneous equations (e.g., by use of the Newton-Raphson method), the values of $\mu_x$ and $\sigma_x^2$ can be estimated from the mean value $\mu_y$ and the variance $\sigma_y^2$ of the measured data $y$ after passing the dynamic range $[a,b]$. Thus, substituting the estimated values of $\mu_x$ and $\sigma_x^2$ into Eq.(4), we can estimate the objective $L_{eq}$ noise evaluation index.

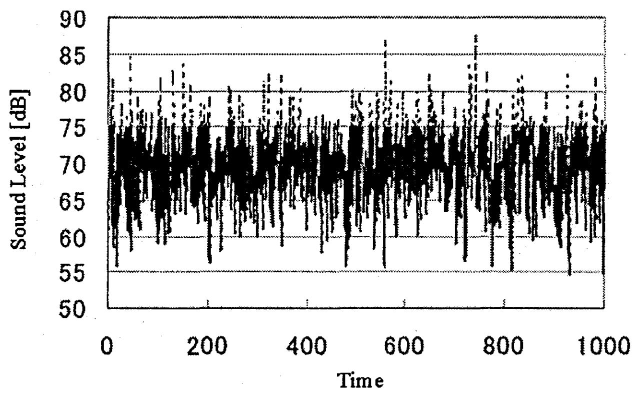
## 3. EXPERIMENTAL CONSIDERATION

### 3.1. Simulation Experiment

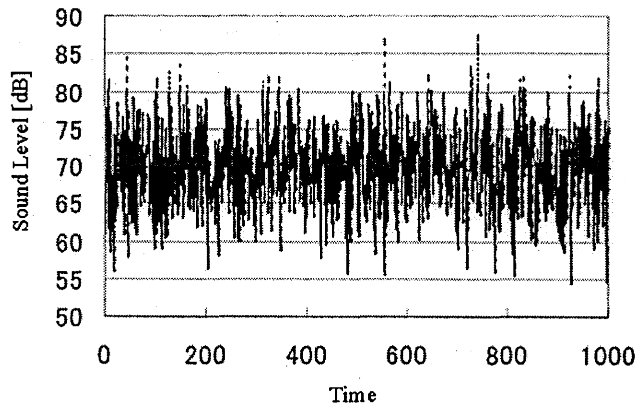
The validity of the proposed method is confirmed by applying it to some simulation experiments. In this simulation experiment, paying our attention to the actual situation of acoustic measurements, we consider the saturation of the sound level fluctuation. First, Gaussian random numbers with mean value of 70.0 [dB] and variance of 25.0 [$dB^2$] have been generated as a simulation data. Next, Gaussian random numbers with mean value of 70.0 [dB] and variance 30 [$dB^2$] have been generated. Let us define the former case as Case A. Let us also define the latter one as Case B. In each case, the saturation levels are selected to 70.0 [dB], 75.0 [dB] and 80.0 [dB]. The generated level fluctuations for each case are shown in Figs.1 and 2. Table 1 shows the estimated results for Case A by using the proposed method. The estimated results for Case B are shown in Table 2. From these results, the estimated results of $L_{eq}$ are in good agreement with the experimental values of $L_{eq}$, as compared with the values evaluated directly from the saturated level data.




(i)Saturation level 70.0 [dB]

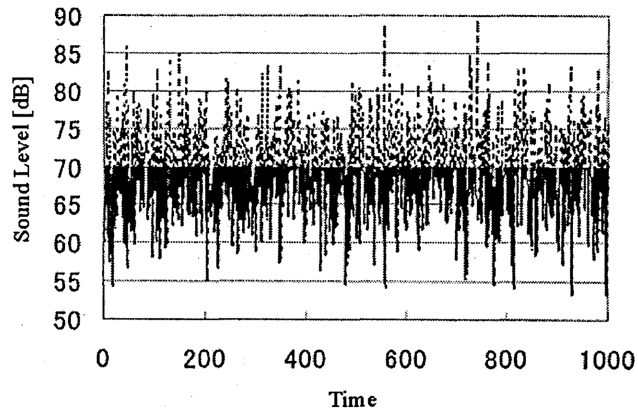



(ii) Saturation level 75.0 [dB]

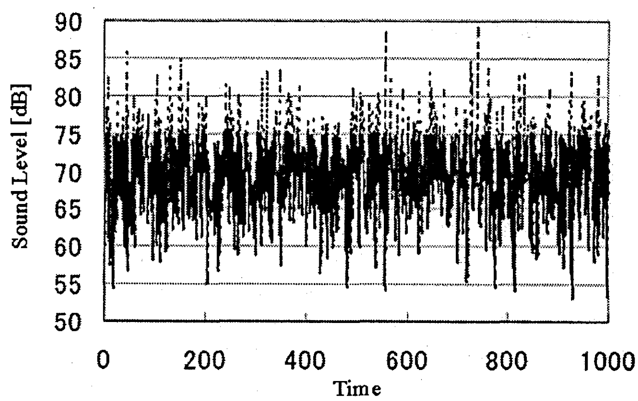



(iii) Saturation level 80.0 [dB]

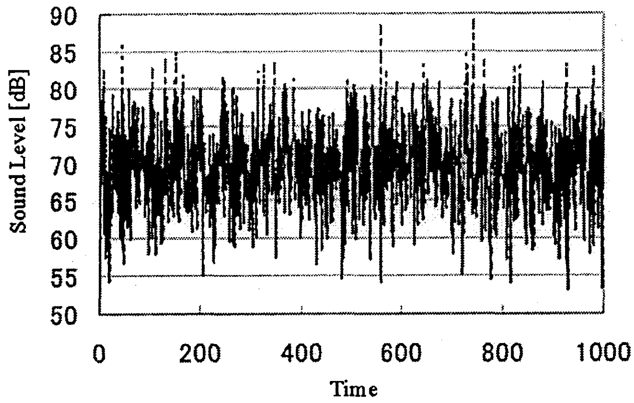
Fig.1 Level fluctuations for each saturation level (Case A) [----- original ; —— saturated].




(i) Saturation level 70.0 [dB]




(ii) Saturation level 75.0 [dB]




(iii) Saturation level 80.0 [dB]

Fig.2 Level fluctuations for each saturation level (Case B) [------ original ; —— saturated].

Table 1 The estimated results by using the proposed method (Case A).

| Saturation Level [dB] | Experimental value of $L_{eq}$ [dB] | Direct evaluation from the saturated data | | Estimated values by using the proposed method | |
|---|---|---|---|---|---|
| | | $L_{eq}$ [dB] | Error [dB] | $L_{eq}$ [dB] | Error [dB] |
| 70.0 | 72.8 | 68.6 | -4.2 | 70.6 | -2.2 |
| 75.0 | | 71.1 | -1.7 | 74.3 | +1.5 |
| 80.0 | | 72.3 | -0.5 | 73.4 | +0.6 |

Table 2 The estimated results by using the proposed method (Case B).

| Saturation Level [dB] | Experimental value of $L_{eq}$ [dB] | Direct evaluation from the saturated data | | Estimated values by using the proposed method | |
|---|---|---|---|---|---|
| | | $L_{eq}$ [dB] | Error [dB] | $L_{eq}$ [dB] | Error [dB] |
| 70.0 | 73.4 | 68.5 | -4.9 | 70.8 | -2.6 |
| 75.0 | | 71.2 | -2.2 | 74.8 | +1.4 |
| 80.0 | | 72.4 | -1.0 | 74.4 | +1.0 |

### 3.2. Application to Road Traffic Noise Data

In this section, we apply the proposed method to the actual road traffic noise data in order to confirm the practical effectiveness of the proposed method. Paying our attention to the actual situation of acoustic measurements, we consider the saturation of the sound level fluctuation. The road traffic noise data have been first measured near a national main road (Route 2) in Fukuyama City by use of a precision sound level meter. Let us define this measurement case as Case C. Second, the road traffic noise data have been measured near a prefectural road (Route 54) in Fukuyama City. Let us define this measurement case as Case D. Third, the road traffic noise data have been measured near a local road (Route 48) in Fukuyama City. Let us define this measurement case as Case E. In order to evaluate precisely and directly the experimental values of $L_{eq}$, the A-weighted sound level fluctuations have been measured at a fine sampling interval of 0.1 [sec]. The measured data of sound level fluctuations for each case are respectively shown in Figs.3, 4 and 5. The frequency distributions for each case are respectively shown in Figs.6, 7 and 8. According to these figures, the frequency distribution for Case E shows apparently non-Gaussian distribution form in comparison with those for Case C and Case D.

The estimated results by using the proposed method for each case are shown in Tables 3, 4 and 5. As shown in Tables 3 and 4, the estimated results are in good agreement with the experimental values of $L_{eq}$. According to Table 5, however, the estimated results for

Case E are not sufficient. This is because of the apparent non-Gaussian property in this case. The proposed method has been derived under the assumption of Gaussian distribution for the sound level fluctuation. Based on the proposed method, it is necessary to consider the above non-Gaussian property of the random phenomena.

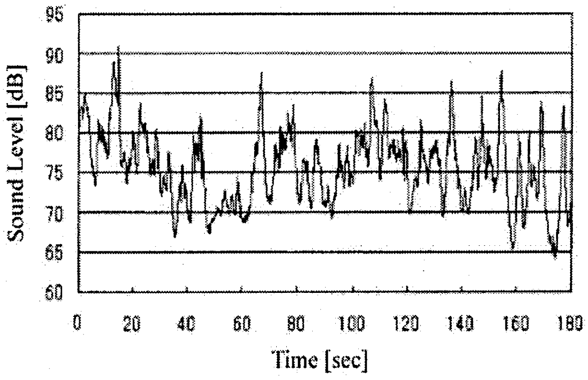



Fig.3 Sound level fluctuation (Case C).

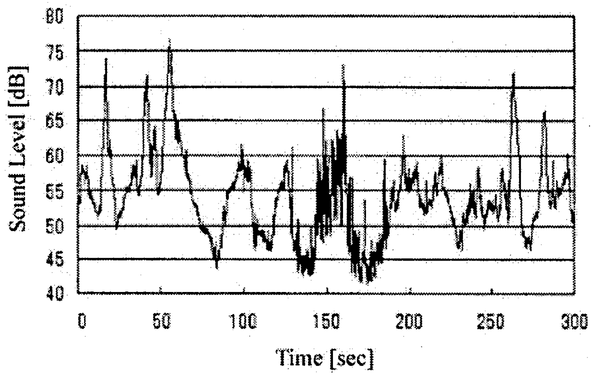



Fig.4 Sound level fluctuation (Case D).

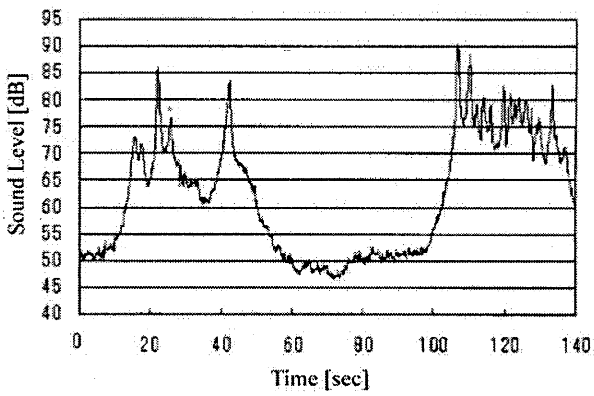



Fig.5 Sound level fluctuation (Case E).

Table 3 The estimated results by using the proposed method (Case C).

| Saturation level [dB] | Experimental value of $L_{eq}$ [dB] | Direct evaluation from the saturated data | | Estimated values by using the proposed method | |
|---|---|---|---|---|---|
| | | $L_{eq}$ [dB] | Error [dB] | $L_{eq}$ [dB] | Error [dB] |
| 76.0 | | 74.6 | -3.5 | 76.3 | -1.8 |
| 78.0 | 78.1 | 75.7 | -2.4 | 78.0 | -0.1 |
| 80.0 | | 76.6 | -1.5 | 79.2 | +1.1 |

Table 4 The estimated results by using the proposed method (Case D).

| Saturation level [dB] | Experimental value of $L_{eq}$ [dB] | Direct evaluation from the saturated data | | Estimated values by using the proposed method | |
|---|---|---|---|---|---|
| | | $L_{eq}$ [dB] | Error [dB] | $L_{eq}$ [dB] | Error [dB] |
| 58.0 | | 54.7 | -5.0 | 57.9 | -1.8 |
| 60.0 | 59.7 | 55.5 | -4.2 | 58.6 | -1.1 |
| 62.0 | | 56.2 | -3.5 | 58.7 | -1.0 |

Table 5 The estimated results by using the proposed method (Case E).

| Saturation level [dB] | Experimental value of $L_{eq}$ [dB] | Direct evaluation from the saturated data | | Estimated values by using the proposed method | |
|---|---|---|---|---|---|
| | | $L_{eq}$ [dB] | Error [dB] | $L_{eq}$ [dB] | Error [dB] |
| 67.5 | 73.4 | 64.1 | -9.3 | 72.8 | -0.6 |
| 70.0 | | 66.0 | -7.4 | 76.5 | +3.1 |
| 72.5 | | 67.8 | -5.6 | 79.6 | +6.2 |

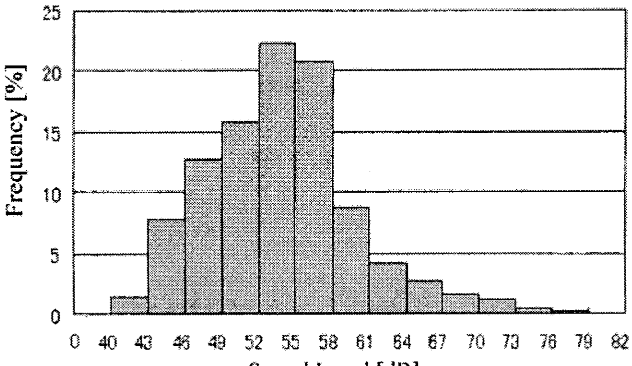



Fig.6 Frequency distribution (Case C).

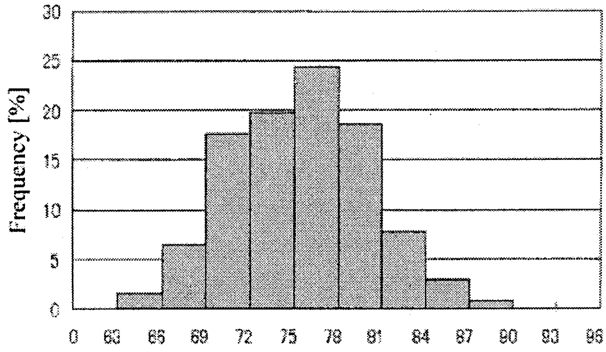



Fig.7 Frequency distribution (Case D).

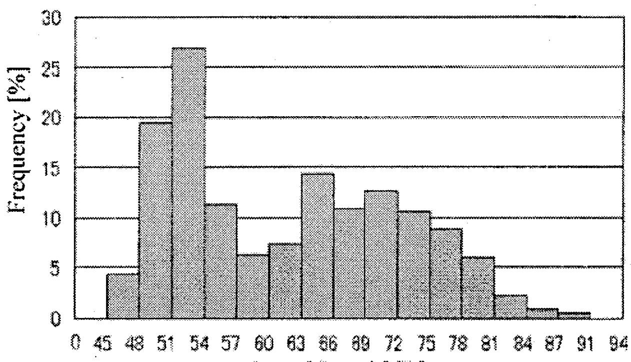



Fig.8 Frequency distribution (Case E).

## 4. CONCLUSION

The noise evaluation index, $L_{eq}$, plays an important role in the field of noise evaluation and/or regulation problems. The sound level fluctuation is often measured within a restricted range due to a dynamic range of measurement instruments in an actual acoustic measurement. Therefore, this incomplete measurement occurs the lack of measurement accuracy of $L_{eq}$.

From the above viewpoint, a signal processing method for the restoration of statistical information on the sound level fluctuation is very important to evaluate $L_{eq}$ by using the above incomplete sound level data. On the other hand, in the previous study, a practical estimation method has been derived under the assumption of a standard Gaussian distribution with the statistical information on the mean value and the variance of the sound level fluctuation.

In this paper, a practical estimation method of $L_{eq}$ has been proposed by considering the measurement mechanism due to a dynamic range. More specifically, an estimation method of the mean and variance of the original sound level fluctuation before passing measurement mechanism with the dynamic range has been first proposed under the assumption of Gaussian distribution of the above original sound level fluctuation. Then, a practical estimation method of $L_{eq}$ has been proposed by using the estimated values of the mean and variance of the original sound level fluctuation.

The validity of the proposed estimation method has been first confirmed through some simulation experiments. Moreover, the practical effectiveness of the proposed method has been experimentally confirmed by applying it to the actual road traffic noise data.

Needless to say, since this research is at an earlier stage of study as a signal processing method of $L_{eq}$ for the actual measurement situation of dynamic range, there remain several problems to be solved in future such as :

(i) As pointed in the experimental consideration, the proposed method has been derived under the assumption of Gaussian distribution for the sound level fluctuation. Based on the proposed method, it is necessary to consider the above non-Gaussian property of the random phenomena.

(ii) It is necessary to apply the proposed method to various kinds of data in many actual environmental noises.

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