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A Practical Estimation Method of L_{eq} Noise Evaluation Index Based on Introduction of Akaike's Information Criterion from Roughly Observed Data

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ABSTRACT

The environmental noise which we encounter in our daily life exhibits various types of probability distribution forms, apart from a standard Gaussian distribution, due to the diversified causes of fluctuation. As is well-known, the noise evaluation index, L_{eq}, plays an important role in the field of noise evaluation and/or regulation problems. Moreover, the noise fluctuation is very often measured in a quantized level form at a discrete time interval. In this paper, a selection method with an optimum order of an expansion type L_{eq} estimation formula, which is generally applicable to arbitrary non-Gaussian level fluctuation, is first proposed by introducing Akaike's information criterion. Based on this L_{eq} estimation formula, a precise evaluation method of L_{eq} is proposed by using the roughly observed data with quantized levels. The effectiveness of the proposed method is experimentally confirmed by applying it to the actual road traffic noise data.

Keywords : L_{eq} noise evaluation index, Akaike's information criterion, quantized level, roughly observed data

1. Introduction

As is well-known, the noise evaluation index, L_{eq}, plays an important role in the field of noise evaluation and/or regulation problems. The environmental noise which we encounter in our daily life exhibits various types of probability distribution forms, apart from a standard Gaussian distribution, due to the diversified causes of

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fluctuation. In the previous paper [1], an explicit L_{eq} estimation formula generally applicable to arbitrary non-Gaussian level fluctuation was proposed by introducing the lower and higher order cumulant statistics in a form of infinite expansion series expression.

In this paper, a selection method of an optimum order of this expansion type L_{eq} estimation formula is first proposed by introducing Akaike's information criterion. On the other hand, in an actual observation, the noise level fluctuation is very often measured in a quantized level form at a discrete time interval [2]. By paying attention to the level-quantization measurement, a precise evaluation method of L_{eq} from the roughly observed data with quantized levels is proposed by using the above estimation formula. In this case, according to the basic concept in the previous studies [3,4], we introduce a general statistical type orthonormal expression of the probability density function for the original noise level fluctuation of continuous level type before passing through the level-quantization measurement mechanism. Next, we propose a reasonable estimation method of the lower and higher order cumulant statistics based on this statistical expression.

The objective L_{eq} can be estimated precisely after substituting the estimated cumulant statistics into the derived L_{eq} estimation formula. The effectiveness of the proposed method is experimentally confirmed by applying it to the actual road traffic noise data.

2. Selection Method of Optimum Order for Expansion Type L_{eq} Evaluation Formula

Let us consider the noise level fluctuation x of an arbitrary non-Gaussian distribution type. As is well-known, the relationship between the noise level fluctuation x and the noise energy fluctuation E is given as follows :

$$x = 10\ell og_{10}\frac{E}{E_0} = M\ln\frac{E}{E_0} \quad (M \equiv 10/\ell n10), \tag{1}$$

where E_0 is the reference noise energy usually taken as $10^{-12}(\mathrm{W/m^2})$. Here, we introduce the moment generating function $M_x(\theta)$ with respect to the noise level fluctuation x, as follows :

$$M_x(\theta) = \langle \exp(\theta M \ln E/E_0) \rangle, \tag{2}$$

where <*> denotes an averaging operation with respect to the random variable *. The mathematical relationship between the arbitrary order cumulant κ_n with respect to x and the moment generating function $M_x(\theta)$ is given by

$$M_x(\theta) = \exp\left(\sum_{n=1}^{\infty}\frac{\kappa_n}{n!}\theta^n\right). \tag{3}$$

By replacing the parameter θ to $1/M$ in Eqs. (2) and (3), the mean value of E can be easily obtained as follows :

$$\langle E \rangle = E_0 \exp\left(\sum_{n=1}^{\infty}\frac{1}{n!}\cdot\frac{\kappa_n}{M^n}\right). \tag{4}$$

Thus, a substitution of Eq.(4) into the definition of L_{eq} yields a general expansion type expression for estimating L_{eq}, as follows [1] :

$$L_{eq} = 10\log_{10}\frac{\langle E \rangle}{E_0}$$

$$= \kappa_1 + \frac{\kappa_2}{2M} + \frac{\kappa_3}{6M^2} + \frac{\kappa_4}{24M^3} + \cdots$$

$$= \mu + 0.115\sigma^2 + 8.84\times10^{-3}\kappa_3 + 5.09\times10^{-4}\kappa_4 + 2.34\times10^{-5}\kappa_5 + \cdots, \quad (5)$$

where $\mu(=\kappa_1)$ and $\sigma^2(=\kappa_2)$ denote the mean value and the variance of x. From Eq.(5), it is possible to generally estimate L_{eq} by reflecting not only lower order cumulants but also higher order cumulants as the correction terms in a hierarchical form. It should be noted that the above estimation formula agrees completely with a well-known simplified estimation formula [5] derived under the assumption of a standard Gaussian distribution as the first approximation :

$$L_{eq} = \mu + 0.115\sigma^2, \quad (6)$$

since higher order cumulants $\kappa_n(n=3,4,\cdots)$ become zero for this special case. Thus, this evaluation method shows a generalized form including the well-known simplified estimation method as a special case. However, it should be noticed that the sample number of actually obtained data is finite. Moreover, since it is impossible to calculate the infinite expansion term in the actual data processing, the finite order must be inevitably employed. Therefore, we must establish a reasonable method for selecting the number of higher order correction terms in Eq.(5). From the practical point of view, we regard the remaining error after employing an appropriate expansion term as the meaningless error information. Therefore, in a case when the finite number of correction terms is employed, it is possible to employ Akaike's information criterion as the evaluation criterion for determining the optimum order of expansion term. In this case, each expansion coefficient of higher order cumulants in Eq.(5) should be estimated in advance by using least squares method. Thus, we can obtain the optimum order so as to minimize the value of *AIC* (Akaike's information criterion), as follows :

$$AIC = N\ell n\hat{\sigma}_e^2 + 2\times(number of\ parameters) \rightarrow \min. \quad (7)$$

Here, N denotes the number of data and $\hat{\sigma}_e^2$ denotes the variance of estimation errors between the true values measured actually and the estimated values by using the evaluation formulae with finite expansion terms.

3. Precise Evaluation Method from Roughly Quantized Level Data

Now, we consider the quantization problem of noise level measurement. Let x be an original noise level fluctuation with an arbitrary probability density function of continuous level type before passing through a level-quantized measurement mechanism. Also, let y be the quantized level value of x after passing through this level-quantized measurement mechanism. In this case, the mathematical relationship between x and y can be expressed by using the function form $f(\bullet)$ of level-quantization mechanism, as follows :

$$y = f(x). \quad (8)$$

In general, a level difference interval is set to be a fixed constant value Δ. Thus, by introducing integer number i, the above relationship can be rewritten as

$$y = i\Delta(i\Delta \le x < (i+1)\Delta). \quad (9)$$

Let us introduce the statistical Hermite expansion type series expression, which is generally applicable to the arbitrary non-Gaussian distribution form, as the probability density function $P(\bullet)$ of the original noise level fluctuation x before passing through the above quantization mechanism, as follows [6] :

$$P(x) = N(x;\mu,\sigma^2)\left\{1+\sum_{n=3}^{\infty}\frac{A_n}{\sqrt{n!}}H_n\left(\frac{x-\mu}{\sigma}\right)\right\} \quad (10)$$

with $$A_n \equiv \frac{1}{\sqrt{n!}}\left\langle H_n\left(\frac{x-\mu}{\sigma}\right)\right\rangle, \quad (11)$$

where $H_n(\bullet)$ denotes the nth order Hermite polynomial. In addition, A_n denotes the expansion coefficient and $N(x;\mu,\sigma^2)$ denotes the well-known Gaussian distribution defined as

$$N(x;\mu,\sigma^2) \equiv \frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}}. \quad (12)$$

It is necessary to estimate the above distribution parameters μ, σ and A_n by using the quantized noise level data y. So, we pay our attention to the fact that the mth order moment statistics $(m = 1,2,\cdots)$ of y can be calculated by $P(x)$, as follows :

$$\begin{aligned}\langle y^m \rangle &= \sum_{i=-\infty}^{\infty}\int_{i\Delta}^{(i+1)\Delta}(i\Delta)^m P(x)dx \\ &= \sum_{i=-\infty}^{\infty}(i\Delta)^m \int_{i\Delta}^{(i+1)\Delta}\frac{1}{\sqrt{2\pi}\sigma}e^{-\frac{(x-\mu)^2}{2\sigma^2}} \\ &\quad \bullet \left\{1+\sum_{n=3}^{\infty}\frac{A_n}{\sqrt{n!}}H_n\left(\frac{x-\mu}{\sigma}\right)\right\}dx. \quad (13)\end{aligned}$$

Therefore, after employing the number of the order $m(m = 1,2,\cdots)$ as the number of unknown parameters μ, σ and $A_n(n = 3,4,\cdots)$ in Eq.(13), we can construct the non-linear simultaneous equations for these unknown parameters. For example, it is possible to solve numerically the non-linear simultaneous equations by use of the Newton-Raphson method. The estimated expansion coefficient $\hat{A}_n(n = 3,4,\cdots)$ can be transformed into each cumulant statistics $\kappa_n(n = 3,4,\cdots)$ for evaluating the objective L_{eq}, as follows :

$$\kappa_3 = \sqrt{6}\sigma^3 A_3,\ \kappa_4 = \sqrt{24}\sigma^4 A_4, \cdots. \quad (14)$$

By substituting the estimated values of $\hat{\mu}$, $\hat{\sigma}$ and $\hat{\kappa}_n(n = 3,4,\cdots)$ into the L_{eq} evaluation formula with the optimum order, we can obtain the objective L_{eq} before the level-quantization of noise fluctuation.

4. Experimental Consideration

In order to confirm the effectiveness of the proposed method, it is applied to the actual road traffic noise data with non-Gaussian property. The road traffic noise data of 100 kinds have been measured at various observation points in Fukuyama City by use of a precision sound level meter. In order to evaluate precisely and directly the experimental values of L_{eq}, the A-weighted noise level fluctuation with a fine level quantization interval of 0.1 dB has been measured at a fine sampling time interval of 0.2 sec. The total measuring time interval has been selected as 10 minutes. The estimated values of *AIC* versus the orders of correction term for the measured road traffic noise are shown in Fig.1. From this figure, the optimum order of correction terms is 2 and the optimum evaluation formula of L_{eq} is obtained as follows :

$$\begin{aligned}L_{eq} &= \mu + 0.115\sigma^2 + 8.836\times10^{-3}\kappa_3 \\ &\quad + 5.086\times10^{-4}\kappa_4. \quad (15)\end{aligned}$$

First, we apply the proposed method to 4 kinds of data selected randomly in the above 100 kinds of measured data (these are defined as Case A, Case B, Case C and Case D). By setting the quantization level to two values of upper and lower levels (with 10 dB level-quantization interval), the estimated results of L_{eq} by using the proposed estimation method are shown in Table 1. In order to confirm the practical effectiveness of the proposed method, it is applied to the other 4 kinds of data measured at the other observation points (these are defined as Case E, Case F, Case G and Case H). The estimated results for these cases are shown in Table 2. According to these tables, the estimated values by using the proposed method are in good agreement with the experimental values.

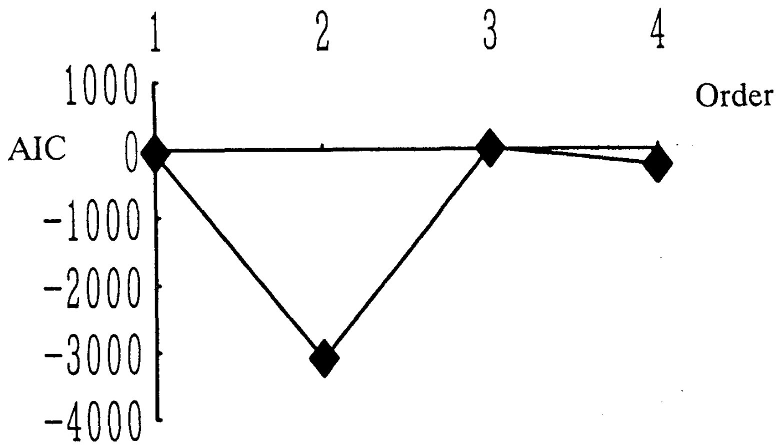

Fig. 1 The values of AIC versus the order of correction term for the measured road traffic noise data.

Table 1 The estimated results for L_{eq} by use of the proposed method.

	Experimental values (dB)	Estimated values (dB)	Estimation errors (dB)
Case A	72.8	72.7	-0.1
Case B	73.2	72.9	-0.3
Case C	74.4	74.7	0.3
Case D	70.6	69.7	-0.9

Table 2 The estimated results for L_{eq} by use of the proposed method.

	Experimental values (dB)	Estimated values (dB)	Estimation errors (dB)
Case E	81.3	81.7	0.4
Case F	81.6	81.6	0.0
Case G	75.7	75.4	-0.3
Case H	75.9	76.0	0.1

5. Conclusion

In the previous paper, an explicit L_{eq} estimation formula generally applicable to arbitrary non-Gaussian level fluctuation was proposed by introducing the lower and higher order cumulant statistics in a form of infinite expansion series expression. In this paper, a selection method of an optimum order of this expansion type L_{eq} estimation formula has been first proposed by introducing Akaike's information criterion. In an actual observation, the noise fluctuation is very often measured in a quantized level form at a discrete time interval. By paying attention to the level-quantization measurement, a precise evaluation method of L_{eq} from the roughly observed data with quantized levels has been proposed by using the above estimation formula. In this case, a general statistical type orthonormal expression of the probability density function for the original noise level fluctuation of continuous level type before passing through the level-quantization measurement mechanism has been introduced. Next, a reasonable estimation method of the lower and higher order cumulant statistics has been proposed based on this statistical expression. The effectiveness of the proposed method has been experimentally confirmed by applying it to the actual road traffic noise data.

There still remain several problems to be solved in future, e.g., applying the proposed method to various kinds of data in many other actual environmental noises and constructing an on-line measurement system based on the proposed method.

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