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A Practical Evaluation Method of L_{eq} Matched to the Restricted Fluctuation Range

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ABSTRACT

The noise evaluation index, L_{eq}, plays an important role in the field of noise evaluation and regulation problems. For the measurement of L_{eq}, it is very convenient to utilize the statistical information on the noise level fluctuation. The well-known evaluation method using the above statistical information was derived under the assumption of a standard Gaussian distribution with the infinite fluctuation range. In an actual noise level fluctuation, however, the fluctuation range is necessarily restricted to some finite value. In this paper, a practical method for evaluating L_{eq} using lower order statistics is proposed by considering matching to above restricted fluctuation range. More specifically, an evaluation method of L_{eq} is derived by the introduction of the Beta distribution matched to the fundamental information on the existence of this finite fluctuation. The effectiveness of the proposed evaluation method is experimentally confirmed by applying it to actual road traffic noise data. Namely, it is shown that the results estimated using the proposed method are in good agreement with the experimental results, compared with the agreement obtained with the results estimated using the usual method derived under the assumption of a standard Gaussian distribution.

Key words: L_{eq} noise evaluation index, Restricted fluctuation range, Environmental noise, Noise level fluctuation

1. Introduction

There are two approaches for measuring the statistics of a stochastic parameter such as L_{eq} (and/or other evaluation indices, e.g., L_x). One is a direct measurement of L_{eq}, giving one of the realistic stochastic value for each phenomenon appearing as a sample process of a population of processes with various fluctuation patterns. Many researchers have already proposed this direct measurement technique according to the original definition of L_{eq} [1]. The other is an indirect method based on consideration of the original population of the various stochastic phenomena under the theoretical background of their stable statistical

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properties. In this case, it is necessary to consider probabilistic approaches toward the objective stochastic phenomena. In this paper, following the latter approach, a practical evaluation method is discussed by the new introduction of the Beta distribution. The basic problems and conceptions of this study are characterized, as follows:

1) In order to evaluate the reliability of L_{eq} measures, one must consider an indirect theoretical method based on consideration of the original population of the various stochastic phenomena, as stated above.
2) According to this way of thinking, several well-known methods [2,3] have been already proposed under the assumption of a standard Gaussian distribution (i.e., using the mean and variance).
3) Although the Beta distribution used here is not familiar to most acoustic engineers, its formulation can simulate the gross shape of the distribution, not only because its distribution edges can be closely related to minimum and maximum values of the stochastic fluctuations, but also the central part of its shape is closely related to the mean and variance.
4) In order to evaluate L_{eq}, it is necessary to obtain many level data with a very fine sampling period, since the noise energy fluctuation after the antilogarithmic transformation of the sampled level datum fluctuates over large range compared to the original decibel-scaled level fluctuation[4].

In relation to the above problems, it is well-known that in the current measurement technique, there still remain essential problems relating to the effect of sampling period on the accuracy of the measurement result. Thus, a fine sampling period in relation to the time constant of the integration, will generally give a good approximation to the results obtained with an idealized true integration. If such a fine sampling time period is not used, the situation becomes different. From the above practical point of view, instead of employing the instantaneous data, it is very convenient to utilize directly the statistical information on the decibel-scaled level fluctuation itself with the aid of a theory for evaluating L_{eq} [5,6] which employs no antilogarithmic transformation. The statistical information then becomes fairly stable and reliable, compared to the instantaneously fluctuating data. In addition, in an actual noise level fluctuation, the fluctuation is necessarily restricted to some finite value. In this paper, a practical method for evaluating L_{eq} using lower order statistics (i.e., mean and variance) is proposed by considering matching to the above restricted fluctuation range of the actual random phenomena. Thus to reflect approximately the information on the two skirts of the distribution and the central part of the random fluctuation, the well-known Beta distribution has been introduced. Finally, the effectiveness of the proposed evaluation method is experimentally confirmed by applying it to several road traffic noise data.

2. Derivation of L_{eq} evaluation method using the Beta distribution

Let us consider the random variable, t, fluctuating within the restricted range [0,1]. As the probability density function, $P_t(t)$, for t, let us introduce the Beta distribution with two distribution parameters, α and γ, as follows [7]:

$$P_t(t)=\frac{1}{B(\gamma,\alpha-\gamma+1)}t^{\gamma-1}(1-t)^{\alpha-\gamma} \tag{1}$$

where B(p,q) denotes the Beta function defined by:

$$B(p,q) \triangleq \int_0^1 t^{p-1}(1-t)^{q-1}dt \tag{2}$$

At this time, the mean, μ_t, and the variance, σ_t^2, are respectively given by [7]:

$$\mu_t = \frac{\gamma}{\alpha+1}$$
$$\sigma_t^2 = \frac{\gamma(\alpha-\gamma+1)}{(\alpha+1)^2(\alpha+2)} \tag{3}$$

Therefore, based on the above Beta distribution, the probability density dunction, $P_x(x)$, for the noise level fluctuation, x, with an arbitrary restricted range $[a,b]$ can be expressed by using the probability measure preserving transformation $[(b-a)t+a]$, as follows:

$$P_x(x) = P_t(t)\left|\frac{dt}{dx}\right|\Bigg|_{t\to x} = \frac{1}{B(\gamma,\alpha-\gamma+1)(b-a)}\left(\frac{x-a}{b-a}\right)^{\gamma-1}\left(1-\frac{x-a}{b-a}\right)^{\alpha-\gamma} \tag{4}$$

where the two distribution parameters α and γ are given in terms of the mean, μ_x, its variance, σ_x^2, for x, the maximum value, b, and the minimum value, a, of the restricted range, as follows:

$$\alpha = \frac{(\mu_x-a)(b-\mu_x)}{\sigma_x^2} - 2$$
$$\gamma = \frac{(\alpha+1)(\mu_x-a)}{b-a} \tag{5}$$

From Eq.(4), the probability density function, $P_x(.)$, for the related noise energy fluctuation, Z ($Z \triangleq E/E_0$; $x = 10\log_{10} E/E_0 = M\ln Z$ ($M \triangleq 10/\ln 10$), E: noise energy fluctuation, E_0: reference noise energy usually taken as 10^{-12} W/m^2), can be obtained by use of the probability measure preserving transformation, as follows:

$$P_Z(Z) = P_x(x)\left|\frac{dx}{dZ}\right|\Bigg|_{x\to Z} = \frac{M}{Z}\frac{1}{B(\gamma,\alpha-\gamma+1)(b-a)}$$
$$\cdot\left(\frac{M\ln Z-a}{b-a}\right)^{\gamma-1}\left(1-\frac{M\ln Z-a}{b-a}\right)^{\alpha-\gamma} \tag{6}$$

Therefore, the mean value of Z, which is closely connected with L_{eq} can be expressed by using Eq.(6), as follows:

$$\langle Z\rangle \triangleq \int_{e^{a/M}}^{e^{b/M}} ZP(Z)dZ = \frac{M}{B(\gamma,\alpha-\gamma+1)(b-a)}\int_{e^{a/M}}^{e^{b/M}}\left(\frac{M\ln Z-a}{b-a}\right)^{\gamma-1}$$
$$\cdot\left(1-\frac{M\ln Z-a}{b-a}\right)^{\alpha-\gamma}dZ \tag{7}$$

Then, after introducing a dimensionless variable $\xi\,[\triangleq(M\ln Z-a)/(b-a)]$, Eq.(7) is reduced to the following simplified expression:

$$<Z>=\frac{e^{a/M}}{B(\gamma,\alpha-\gamma+1)}\int_0^1 \xi^{\gamma-1}(1-\xi)^{\alpha-\gamma}e^{(b-a)\xi/M}d\xi \tag{8}$$

Furthermore, by carrying out termwise integrations after applying a Taylor's expansion to the exponential term in Eq.(8), one can derive the following expression:

$$<Z>=e^{a/M}F(\gamma,\alpha+1;\frac{b-a}{M}) \tag{9}$$

where $F(\gamma, \alpha; \zeta)$ denotes a degenerate hypergeometric function defined by [8]:

$$F(\gamma,\alpha;\xi)\triangleq\sum_{n=0}^{\infty}\frac{\gamma(\gamma+1)\cdots(\gamma+n-1)}{\alpha(\alpha+1)\cdots(\alpha+n-1)}\frac{\xi^n}{n!} \tag{10}$$

In the actual calculation of this function, it is possible to employ the following expression on the usual Gamma function:

$$F(\gamma,\alpha;\xi)=\frac{F(\alpha)}{\Gamma(\gamma)}\sum_{n=0}^{\infty}\frac{\Gamma(\gamma+n)}{\Gamma(\alpha+n)}\frac{\xi^n}{n!} \tag{11}$$

Consequently, substituting Eq.(9) into the definition of L_{eq}, an explicit expression for evaluating this noise evaluation index is derived as follows:

$$L_{eq}\triangleq 10\log_{10}<Z>=a+M\ln F(\gamma,\alpha;\frac{b-a}{M}) \tag{12}$$

3. Experimental work

In this section, the effectiveness of the proposed evaluation method is experimentally confirmed and compared with the well-known simplified evaluation method derived under the assumption of a standard Gaussian distribution[2]:

$$L_{eq}=\mu_x+0.115\sigma_x^2 \tag{13}$$

As one of the typical examples of environmental noise, three kinds of road traffic noise were employed in this experimental work. They have been respectively measured in the following ways: —Case A, near a national main road (traffic volume: 263 vehicles per ten minutes); Case B, near a country road (traffic volume: 38 vehicles per ten minutes); Case C, also near a country road (traffic volume: 41 vehicles per ten minutes). The actual situations for the measurement of road traffic noise for each case are respectively shown in Figs.1,2 and 3. The measuring period and the sampling interval for each case have been selected as 10 minutes and 1 second respectively. The measurement system based on the proposed method has been constructed by use of a digital sound level meter and a portable microcomputer with an RS-232C type interface.

Table 1 shows the results calculated by use of the proposed evaluation method (see Eq.(12)), and well-known simplified evaluation method (see Eq.(13)). According to this table, the values calculated by using the proposed method are in good agreement with the

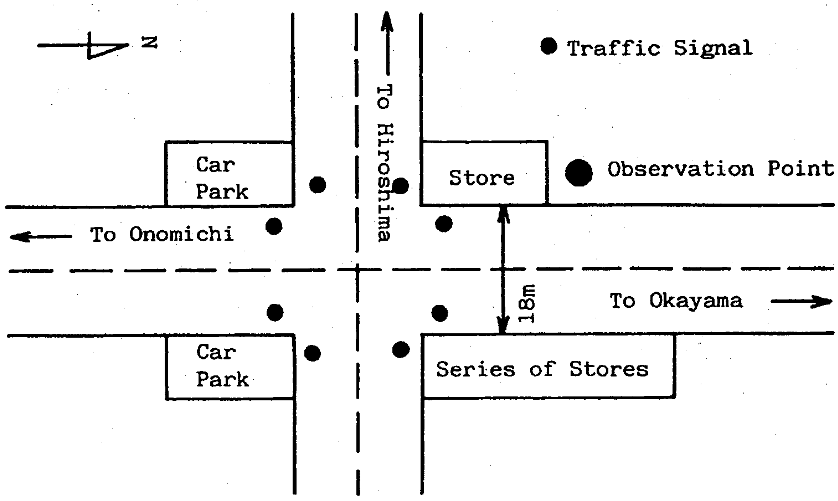

Fig.1 Actual situation for the measurement of road traffic noise (Case A).

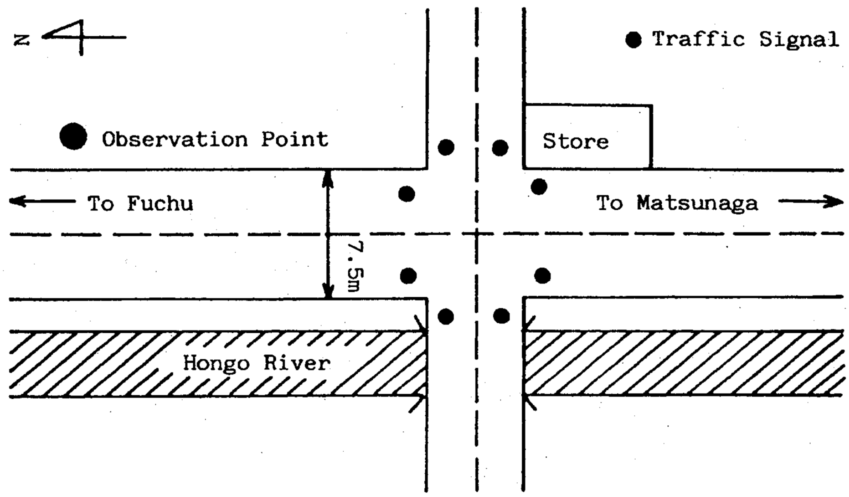

Fig.2 Actual situation for the measurement of road traffic noise (Case B).

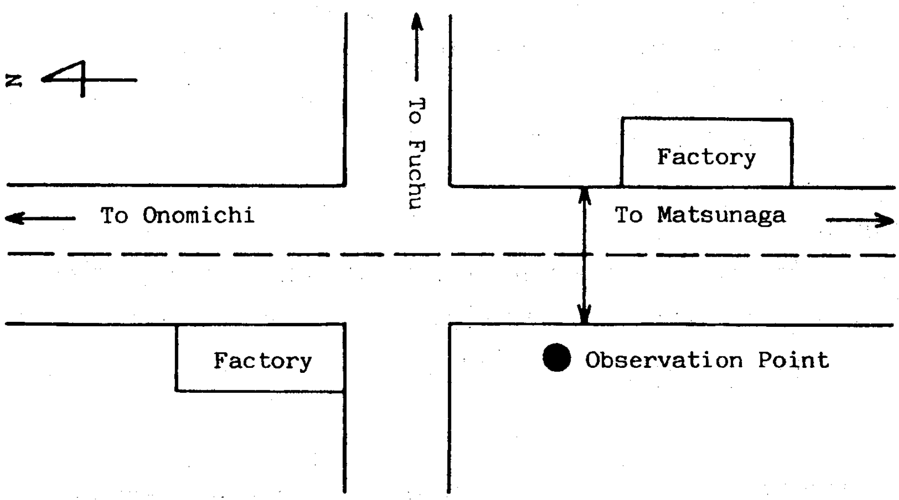

Fig.3 Actual situation for the measurement of road traffic noise (Case C).

actual value observed by using a precision integration sound level meter for L_{eq}, because it is obtained by its original definition and so it is reasonable and practical to employ its value as the standard for comparison. (Of course, a slight measurement error would be still seen even when using a precision meter.)

Table 1 The calculated results for the L_{eq} evaluation index using the proposed evaluaton method.

Case	Observed value of L_{eq} [dBA]	Estimated value by use of Eq.(13) [dBA]	Estimated value by use of the proposed method [dBA]
A	75.9	76.4	76.1
B	73.5	71.5	72.8
C	70.7	67.7	69.6

4. Conclusion

In this paper, a practical method for determining an L_{eq} noise evaluation index has been proposed by newly introducing Beta distribution matched to the most fundamental information on the existence of a restricted range in the actual noise level fluctuation. The effectiveness of the proposed evaluation method has been experimentally confirmed by applying it to actual road traffic noise data.

Of course, this study represents an early stage of study and so it has been focussed only on the fundamental aspects. Accordingly, there still remain many future problems, as follows:

1) This method must be applied to many other actual cases to broaden and confirm its practical effectiveness.
2) For the purpose of evaluating more precisely the objective L_{eq} noise evaluation index, it is possible to generalize systematically this fundamental method by introducing a general explicit formula of statistical Jacobi expansion type [9] based on not only the lower order statistical information but also the higher order statistical information.
3) Based on this method, a practical on−line method for the sequential observation of L_{eq} can be proposed using a microcomputer.

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