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Motion of Particle in Steady Flow by Numerical Experiment

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ABSTRACT

The motion of the tracer particle immersed in flow is seemed to be an interesting problem in the flow visualization. Its motion can be described by the B.B.O.'s equation. In this paper the method of successive approximation is used to solve the B.B.O.'s equation. In order to visualize the theoretical results of the motion of the tracer particle, the numerical experiments are tried in the steady eddy flow of the wake behind a square pole. The effect of the Basset term becomes clear and the characteristics of the motion of particle is presented.

Keywords: B.B.O.'s equation, particle motion, tracer particle, numerical experiment

1. Introduction

In the method of flow visualization, many kinds of particle are used as the tracer of the fluid flow. The motion of the tracer particle immersed in flow is seemed to be an interesting problem in the measurement of fluid flow.

Its motion is represented by the B.B.O.'s (Basset, Boussinesq, Oseen) equation. Historically, many analytical methods, for example, Fourier, Laplace transform, theorem of Abel, were applied to solve the B.B.O.'s equation.[1),2),3)]

The Basset term[4)] of the B.B.O.'s equation takes account of the effect of the deviation in flow pattern from steady state. When the particle is accelerated at a high rate by a strong external force, the Basset term may become substantial, increasing the instantaneous coefficient of resistance to many times its value at steady state.

In this paper, the method of successive approximation is used to solve the B.B.O.'s equation. On basis of the theoretical analysis, the trajectory of particles is examined in the steady wake flow behind a square pole by numerical experiment.

2. Theoretical Consideration

When the characteristics of flow ($Re<0.4$, $Re=2r(U_f-U_p)/\nu$) is completely well known,

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that means the distribution of velocity, pressure, and density is known in the entire region of the flow, the motion of a solid spherical particle immersed in flow can be described as the following well known B.B.O.'s equation (1).

$$\underset{(I)}{M_p dU_p/dt} = \underset{(II)}{6\mu\pi r V} + \underset{(III)}{M_f dU_f/dt} + \underset{(IV)}{1/2 M_f dV/dt} + \underset{(V)}{6r^2\sqrt{\pi\rho_f\mu}\int_0^t (dV/d\tau/\sqrt{t-\tau})d\tau} - \underset{(VI)}{(M_p - M_f)g} \tag{1}$$

Where, $V=U_f-U_p$. The suffix f, p indicate the quantities concerning with fluid and particle, and the symbol of M is mass; ρ, density; U, velocity; μ, dynamic viscosity of fluid; r, radius of particle; g, gravity acceleration.

The forces of each term of the equation (1) are (I) inertia force, (II) friction force, (III) pressure gradient, (VI) force to accelerate the apparent mass of the particle, (V) Basset term, and (VI) buoyancy force.

In this paper, the method of successive approximation[5] is used to solve the equation (1). This equation can be normalized by using reference velocity U_s and time T, where $U_s=2r^2 \times (\sigma-1)g/9\nu$, $T=2r^2(\nu+1/2)/9\nu$. As the result, the equation (1) is reformed to the equation (2).

$$U_* = f_* + \int_0^t K\cdot U_* d\tau_* \tag{2}$$

Where, $f_*=1+\gamma dU_{f*}/dt_* - V_{*0}$; $K=-(1+\lambda/\sqrt{t_*-\tau_*})$. And the other related values are $U_*=dV_*/dt_*$; $V_*=V/U_s=\int_0^t U_* d\tau_* + V_{*0}$; $V_{*0}=V_*(0)$; $U_{f*}=U_f/U_s$; $\gamma=(\sigma-1)/(\sigma+1/2)$; $t_*=t/T$; $\sigma=M_p/M_f$; $\lambda=\sqrt{9/2\pi(\sigma+1/2)}$

When the successive approximation is applied to solve the integral equation (2), the equation (3) can be obtained

$$U_* = f_* + \int_0^t H\cdot f_* d\tau_* \tag{3}$$

where, $H=\sum_{n=0}^{\infty}(-1)^n(t_*-\tau_*)^n/n! - \sum_{n=0}^{\infty}\sum_{k=0}^{n}(-1)^{n+k}(2^n\lambda/(2n-1)!!)(t_*-\tau_*)^{(2n-1)/2}$

$$\times {}_{n+1+k}C_{2k+1}(\lambda^2\pi)^k + \sum_{n=0}^{\infty}\sum_{k=0}^{n}(-1)^{n+k}(t_*-\tau_*)^n/n!\ {}_{n+2+k}C_{2k+2}(\lambda^2\pi)^{k+1}$$

In the equation (3), the function f_* is a known function which is included buoyancy force, flow acceleration, and initial mutual velocity of the particle.

The motion of particle is discussed[6] mainly based on the equation (3).

(i) Superposition

The superposition of the solutions can be possible in the equation (2) and (3).

(ii) Velocity of particle

The normalized velocity of the particle can be obtained to integrate the equation (3).

$$U_{p*} = U_{f*} - \int_0^{t*}(f_* + \int_0^{t*} H\cdot f_* d\tau_*)d\tau_* - V_{*0} \tag{4}$$

where, $V_{*0}=Uf_*(0)-U_{p*}(0)$.

(iii) Basset term

The Basset term (Eq. (1), (V)) is represented the term of $-\lambda/\sqrt{t_*-\tau_*}$ in kernel K function of the equation (2). When the Basset term is neglected from the equation (1), then the kernel K=-1 in the equation (2), the function H in the equation (3) is calculated $H=-\exp-(t_*-\tau_*)$. This is the same result that the second and the third terms are neglected from the H function of the equation (3). The function H is shown in **Fig. 1**.

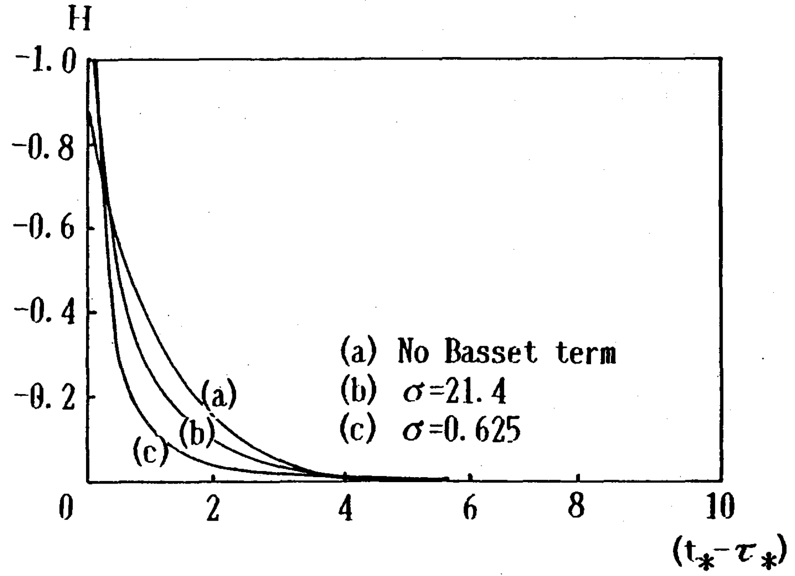


Fig. 1 H function

(iv) Simple initial condition of particle

(a) Particle is released from fixed to free situation in flow at $t_*=0$, $U_{p*}(0)=0; V_{*0}=Uf_*$.

(b) Particle is immersed enough long time in steady flow before $t_*=0$, $U_{p*}(0)=Uf_*(0)$; $V_{*0}=0$.

(v) Equal density between fluid and particle

When the density of particle is the same of fluid, the reference velocity $U_s=0$. In this case, the reference velocity U_s is selected $U_s=2r^2g/9\nu$. And rewriting the equation (1), the obtaining equations are the same equation (2) ~ (4).

However, the function f_* is $f_*=-V_{*0}$.

The motion of the particle is affected only initial velocity of the particle.

(vi) Particle velocity in uniform flow neglected gravity force

When the fixed particle is released in the uniform flow at $t_*=0$, the velocity of the particle $U_{p*}(t_*)$ is

$$U_{p*}/Uf_* = \int_0^{t_*} d\tau_* + \iint_0^{t_*} H d\tau_* \tag{5}$$

The time variation of the particle velocity is shown in **Fig. 2**. We can obtain similarly the vertical velocity of the particle ($U_{p*}=0$ at $t_*=0$) in the still fluid with gravity force.

$$U_{p*} = -\left(\int_0^{t_*} d\tau_* + \iint_0^{t_*} H d\tau_*\right) \tag{6}$$

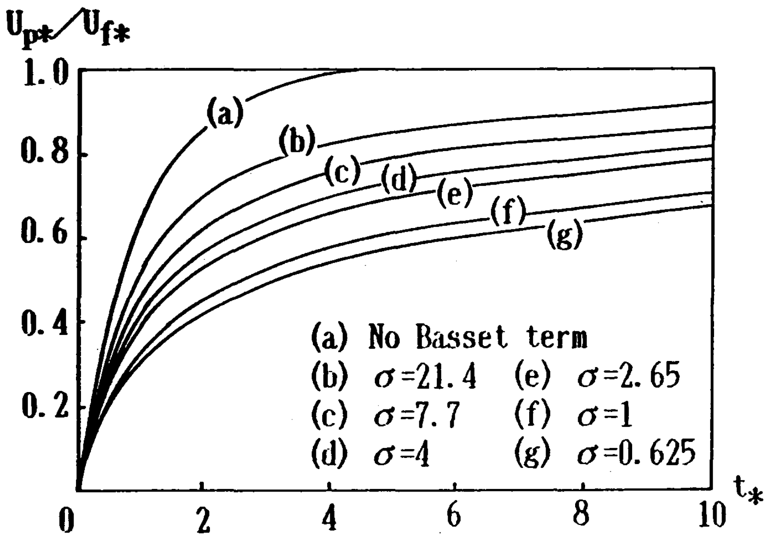


Fig. 2 Velocity of particle

(vii) Velocity of particle in uniformly accelerated flow neglected gravity force

When the particle ($U_{p_*}=U_{f_*}$ at $t_*<0$) flows in the uniformly accelerated flow ($dU_{f_*}/dt_*=$ constant) at $t_*=0$, the velocity of the partcile $U_{p_*}(t_*)$ is

$$U_{p_*}=U_{f_*}-\gamma dU_{f_*}/dt_*(\int_0^{t*}d\tau_*+\iint_0^{t*}Hd\tau_*) \tag{7}$$

The final velocity of the particle approaches $U_{p_*}=U_{f_*}-\gamma dU_{f_*}/dt_*$ at $t_*\cong\infty$.

In the vertical uniform flow with the flow velocity U_{f_*} and gravity force, the particle has the final settling velocity $U_{p_*}=U_{f_*}-1$ similarly.

The coefficient of the equation (7) is the related value of the density of the particle ($\gamma=(\sigma-1)/(\sigma+1/2)$). Therefore, the velocity of the particle is increased or decreased depending on the density of the particle ($\sigma\gtreqless1$) and the acceleration of the flow ($dU_{f_*}/dt_*\gtreqless0$).

Let's consider a part of eddy flow refering with **Fig. 3**, the velocity component of x-direction of the flow increases but the y component decreases in this case. When the density of the particle $\sigma>1$, the x component of the velocity of the tracer particle is decreased and the y component is increased in this flow.

The pathline of the particle is located inside or outside of the stream line depending

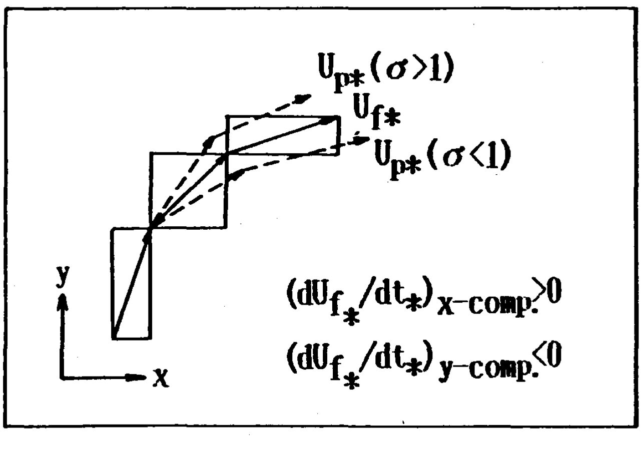


Fig. 3 Schematic motion of particle in accelerated flow

on the density of the particle $\sigma \gtrless 1$ in this flow.

When the settling velocity U_s of the particle is selected enough small, the term of the acceleration $(\gamma dU_{f_*}/dt_*)$ of the equation (7) can be neglected.

3. Numerical Experiment

In order to visualize the theoretical results of the motion of the tracer particle, the numerical experiments of the particle motion are tried in the steady eddy flow of the wake behind a square pole.

(i) Horizontal section without gravity force

In the horizontal section of the flow, the velocity vector of the eddy flow of the wake is calculated numerically as shown in **Fig. 4**. The gravity force is neglected in this calculation. In this case the boundary velocity U_0, the coefficient of kinematic viscosity ν, and the length of square pole 1 are $U_0=15$(cm/sec), $\nu=0.15$(cm^2/sec) and 1=1.4(cm), respectively.

The standard pathline of the particle seems to be the line of the particle having the same density of the fluid ($\sigma=1$, referring 2. (v)). The calculated pathline is shown in **Fig. 5** (a).

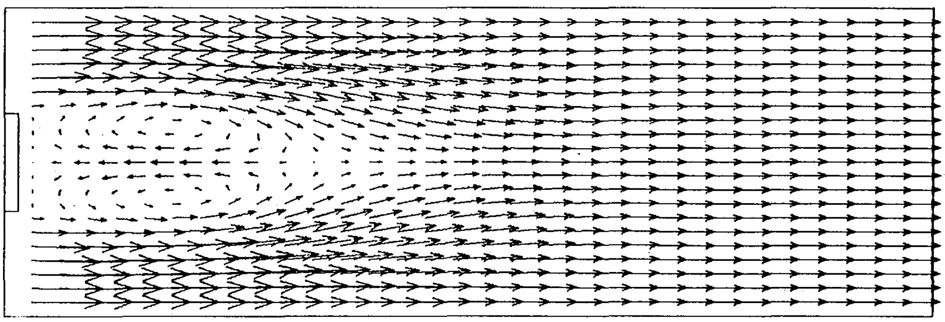


Fig. 4 Flow pattern Re=140 (vertical section)

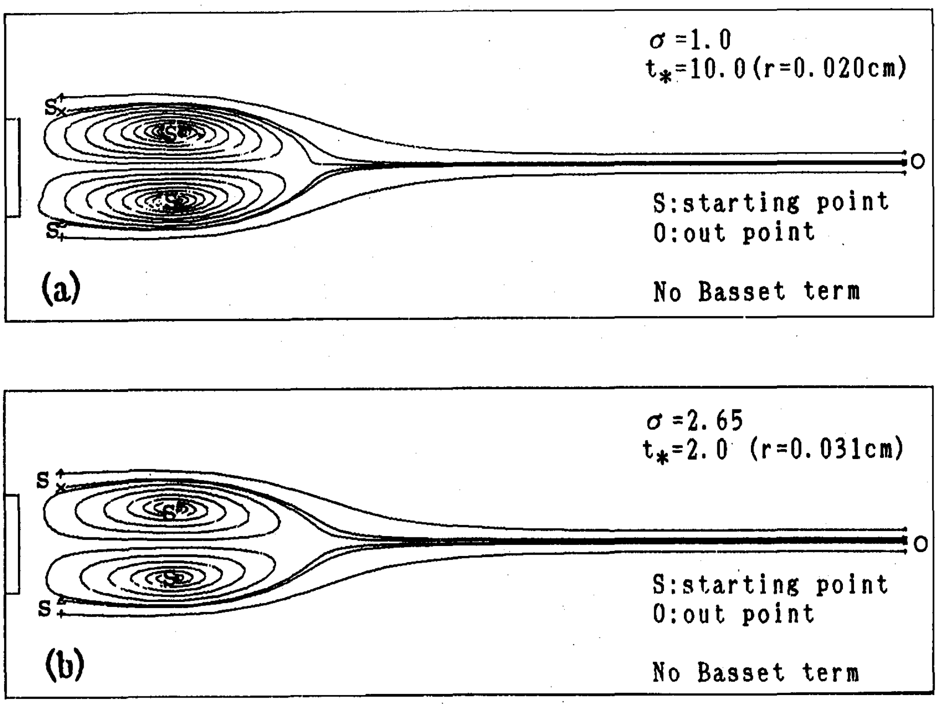


Fig. 5 Trajectory of particles (horizontal section)

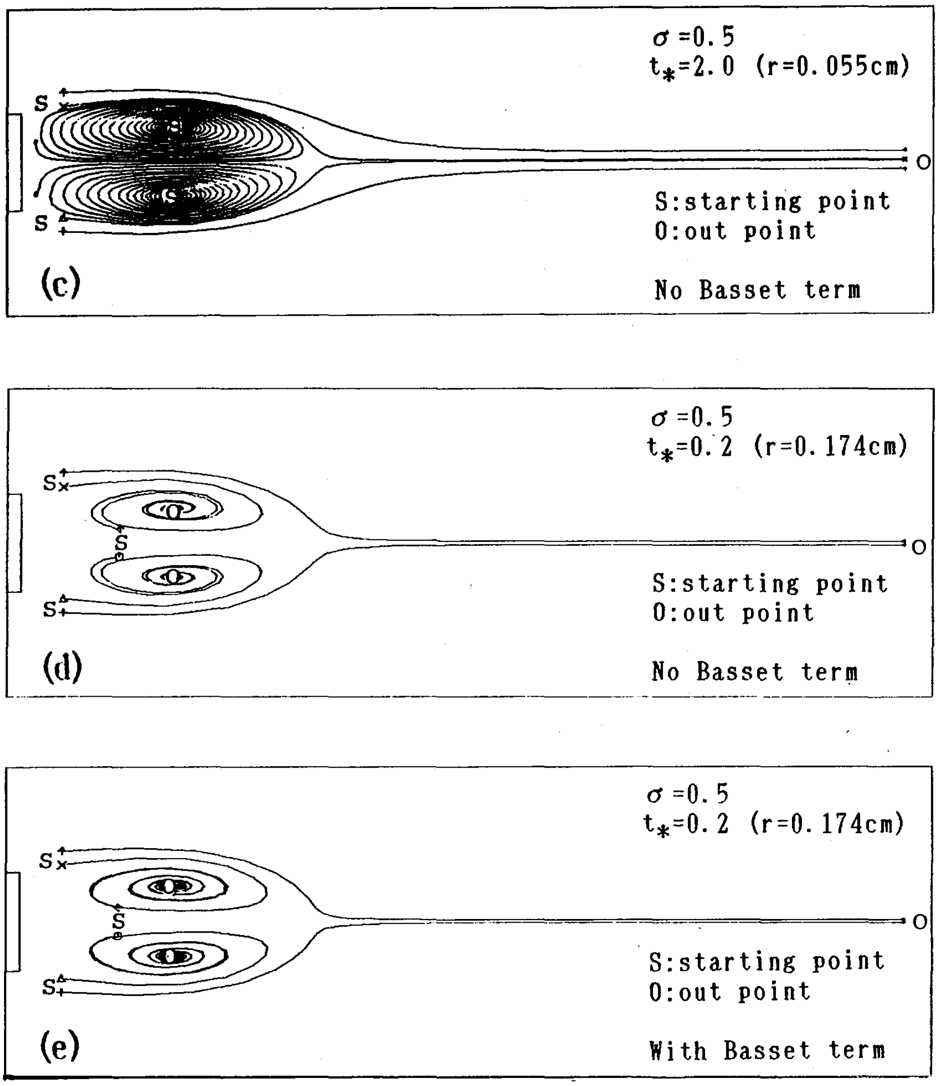


Fig. 5 Trajectory of particles (horizontal section)

In the calculation of this pathline, in order to eliminate the effect of the initial condition of the particles, the unit time t of the calculation is selected as small as possible. Its value is t=0.009(sec). The effect of the Basset term is not remarkable in this calculation. The results of the both calculations with Basset and without Basset are almost the same. The eddy flow seems to be the source type flow. The fluid flows out from inside to outside of the eddy.

The pathlines of the particles having different specific weight ($\sigma \gtrless 1$) and radius are calculated in order to compare the motion of the particle.

In the case of the density of the particle is greater than the fluid (2.65), the pathline of the particle dropped in the center of the eddy flow represents the spiral out trajectory as shown in **Fig. 5(b)**.

When the particle having the smaller density as compared with the fluid ($\sigma=0.5$) is dropped in the center of the eddy flow, the pathline of the particle represents the similar spiral out motion as shown in **Fig. 5(c)**.

However, using the particle with the same density ($\sigma=0.5$) but having large radius, the pathline of the particle represents the spiral in motion as shown in **Fig. 5(d)**. Also the effect

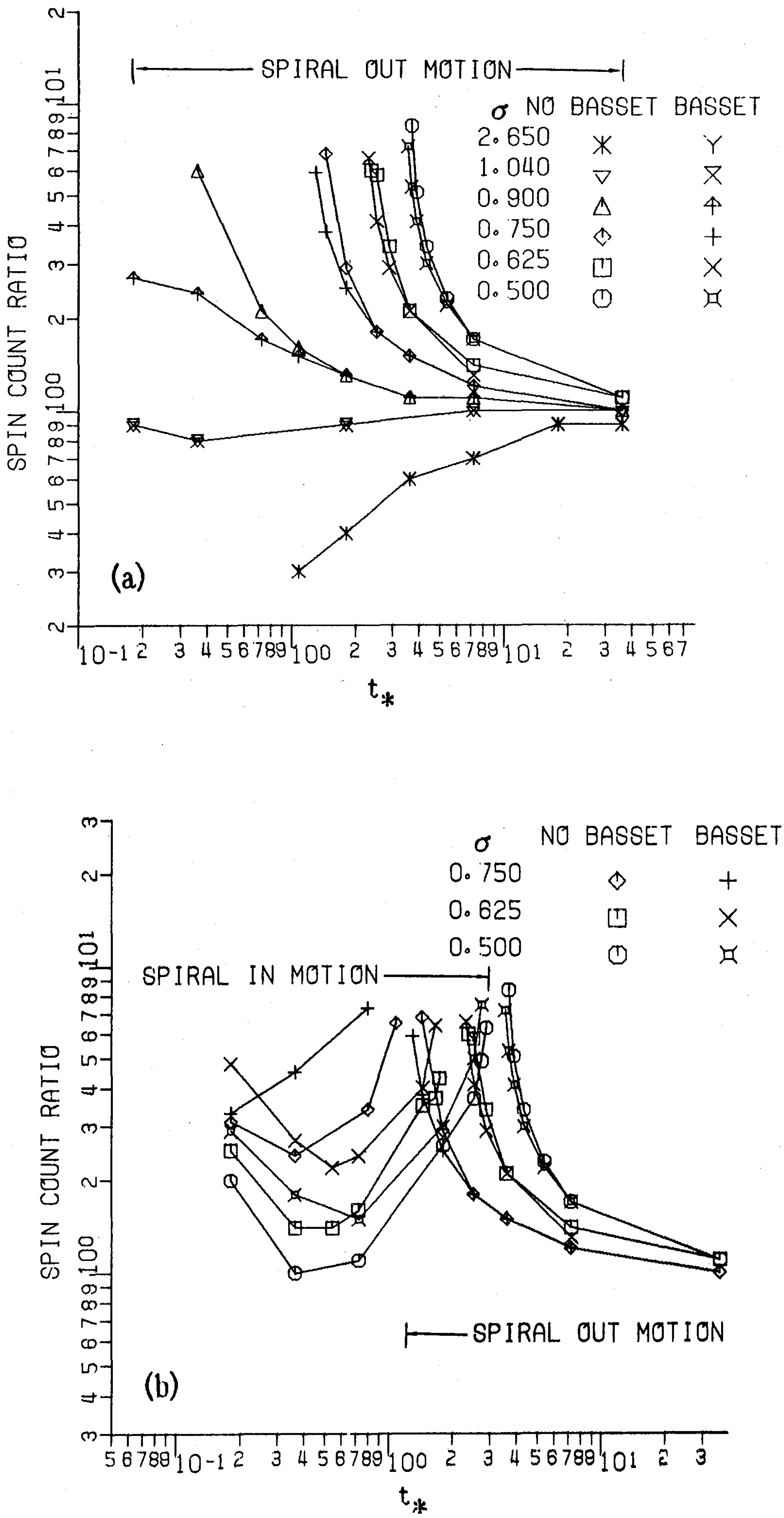


Fig. 6 Spin count of the motion of particle (horizontal section)

of the Basset term can be obtained as compared with the results of **Fig. 5 (e)** and **5 (d)**.

We can confirm the theoretical consideration (2. (vii)) from these numerical experiments. **Fig. 6** shows the relation between t_* and the spin count ratio. The spin count ratio is the value divided the spin count of the spiral motion by the one of the standard pathline. The spiral in or out motion of the particle is related to the radius and the density of the particle.

(ii) Vertical section with gravity force

In the vertical section of the flow, the velocity vector in the wake behind a square pole is calculated numerically as shown in **Fig. 7**. The gravity force is considered in this calculation.

The pathline of the particle with the same density of the fluid is shown in **Fig. 8(a)**. The fluid flows out from inside to outside of the eddy flow.

The all calculated pathlines of the particle having different density and radius represent the spiral out motion as shown in **Fig. 8(b)**, **(c)** and **(d)**.

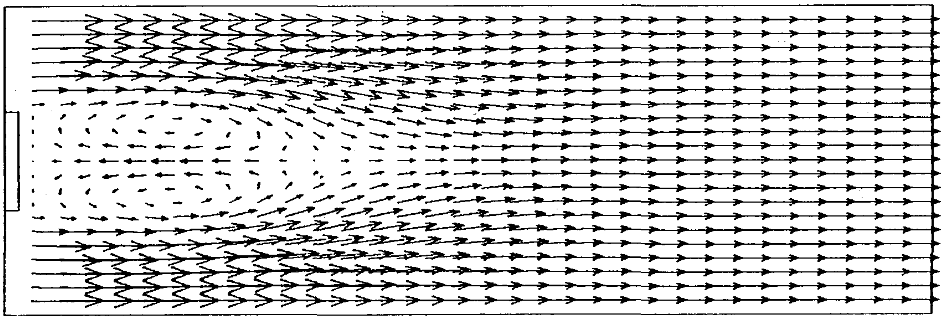
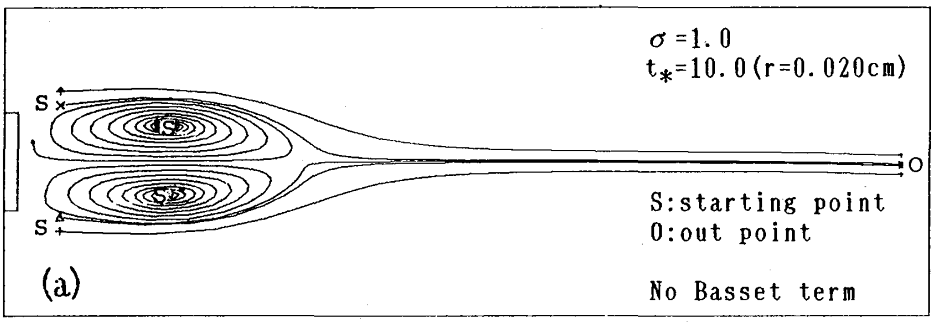


Fig. 7 Flow pattern Re=140 (vertical section)



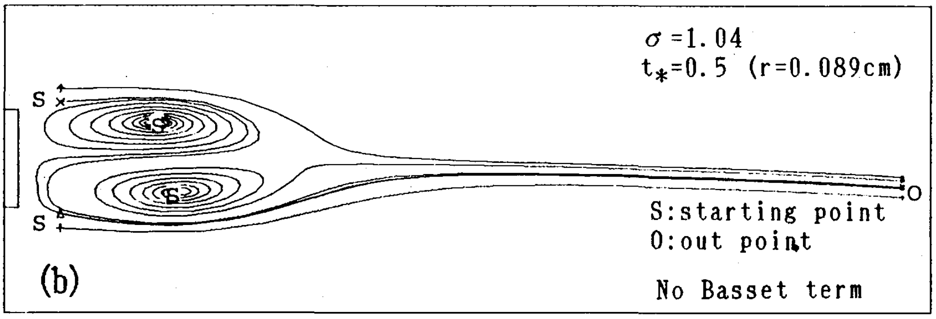
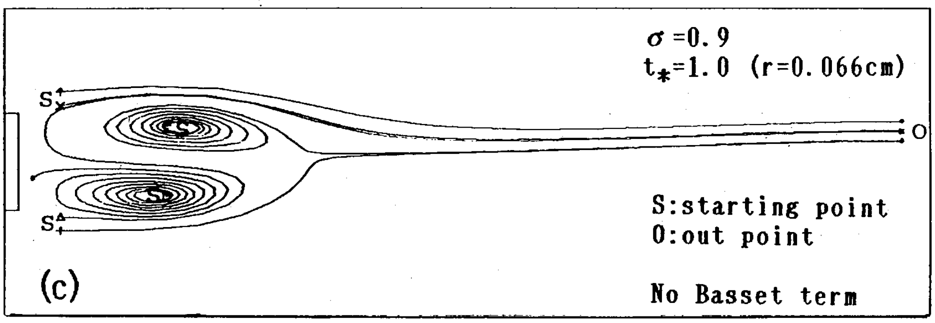


Fig. 8 Trajectory of particles (vertical section)



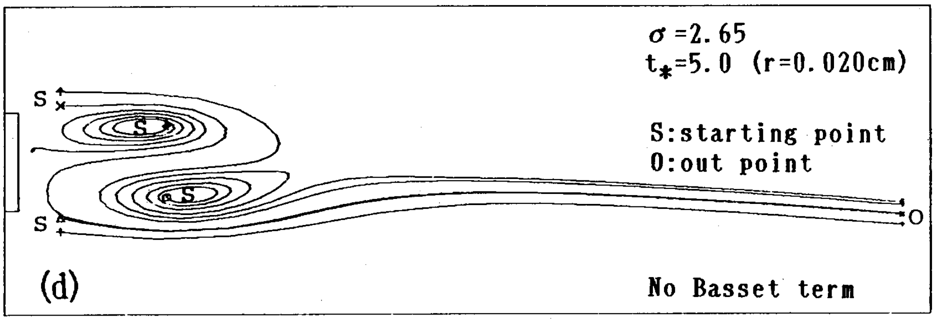


Fig. 8 Trajectory of particles (vertical section)

The existence of the gravity acceleration seems to be the reason of no occurrence of the spiral in motion of the particle.

On comparing with each pathline near the center of the spiral, the figures of pathlines are different. Each pathline compared with the standard pathline is shown in **Fig. 9**. The trajectory of particles seems to be under influences of the gravity force and the size of the particle.

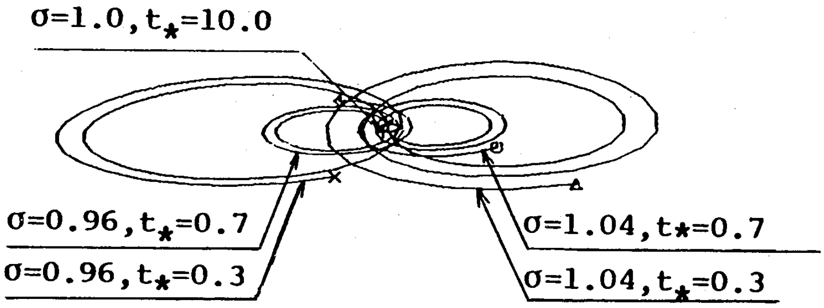


Fig. 9 Comparison with pathlines near the center of spin motion

In order to compare with the spiral motion of particles in the same manner as the horizontal section, we investigate the relation between the spiral motion of the particle and t_*. The spin count ratio changes by the difference in the size and the density of particles as shown in **Fig. 10**.

When we arrange the result of the spin count ratio and t_*, the relation between the spin count ratio and the settling velocity U_S is obtained as shown in **Fig. 11**. The spin count ratios of the particles with the heavy specific weight ($\sigma>1$) and the light one ($\sigma<1$) overlap respectively.

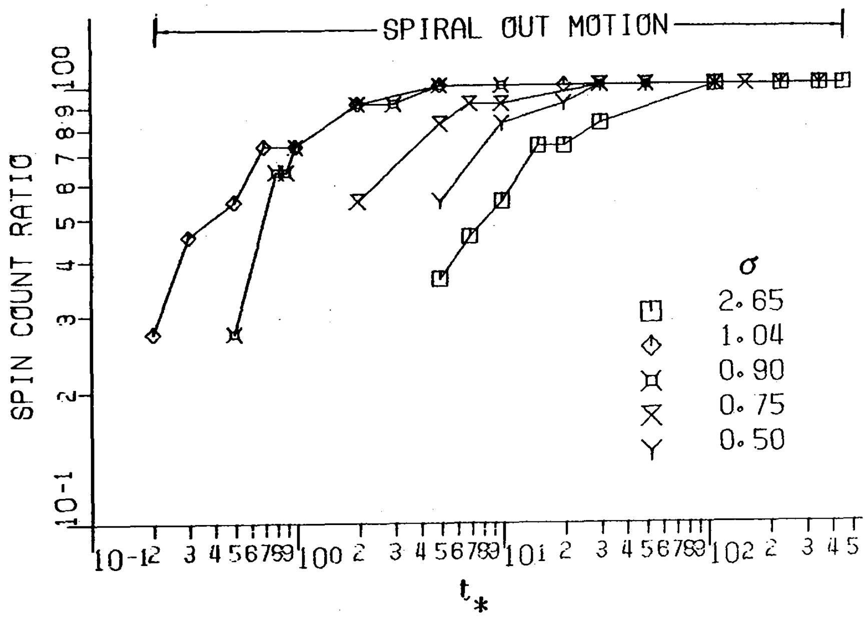


Fig. 10 Spin count of the motion of particle (vertical section)

(iii) Vertical section of upward flow

In order to confirm the Basset term and the relation as shown in **Fig. 11**, numerical experiment of the particle motion is tried in the upward flow. The velocity vector and the standard pathline of the particle (σ=1) are shown in **Fig. 12** and **13(a)**, respectively. The fluid flows out from inside to outside of the eddy flow, and the all calculated pathlines of the particle having different density and radius represent the spiral out motion such as 3.(ii). Several examples are shown in **Fig. 13(b)**, **(c)**, **(d)** and **(e)**.

In the case of the particle which the specific weight is greater than the fluid (σ=2.65), the pathline is traced downward by the influence of the weight of the particle as shown in

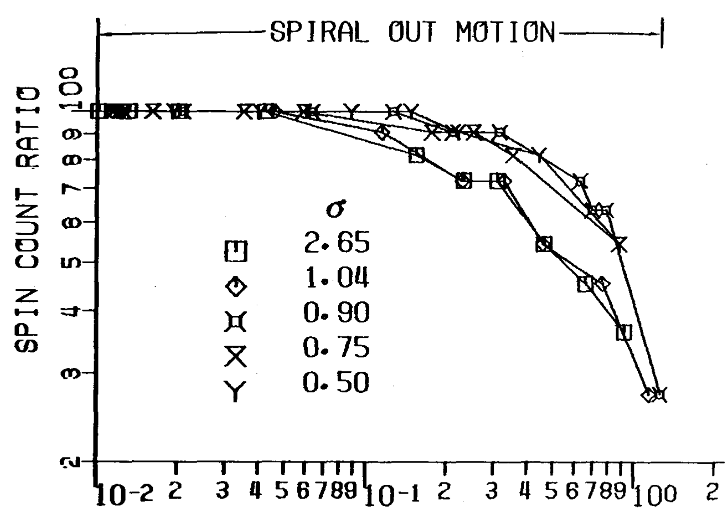


Fig. 11 Relation between $|U_s|$ and spin count ratio (vertical section)

Fig. 13(f). On the other hand, the pathline of the light particle (σ=0.5) with the comparative large radius is traced directly upward as shown in Fig. 13(g).

In such pathlines of particles with the large radius, there are remarkable differences of pathlines by the presence of the Basset term. For example, on comparing Fig. 13(h) with Fig. 13(i), the difference of pathlines is confirmed.

In the same way described above, the relations among the spiral motion, t_* and U_s are shown in Fig. 14 and Fig. 15, respectively. When the absolute value of U_s is greater than 0.6, the deviation of the spin count ratio is produced by the difference of the specific weight.

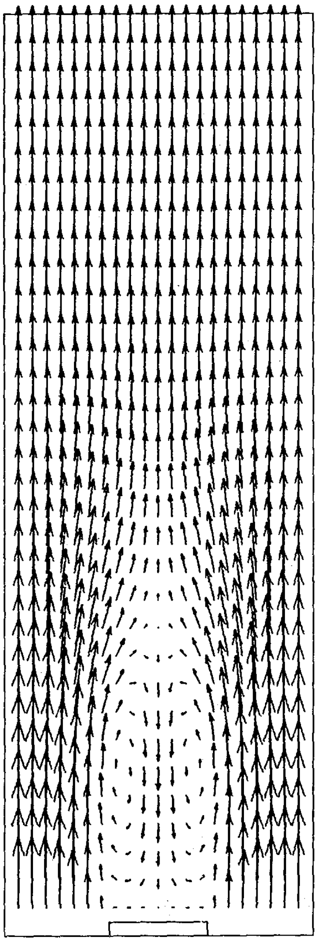


Fig. 12 Flow pattern
Re=140
(upward flow)

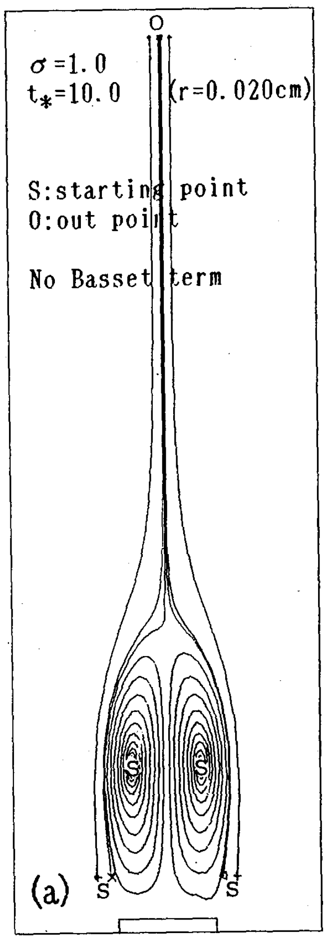


Fig. 13 Trajectory of particles
(upward flow)

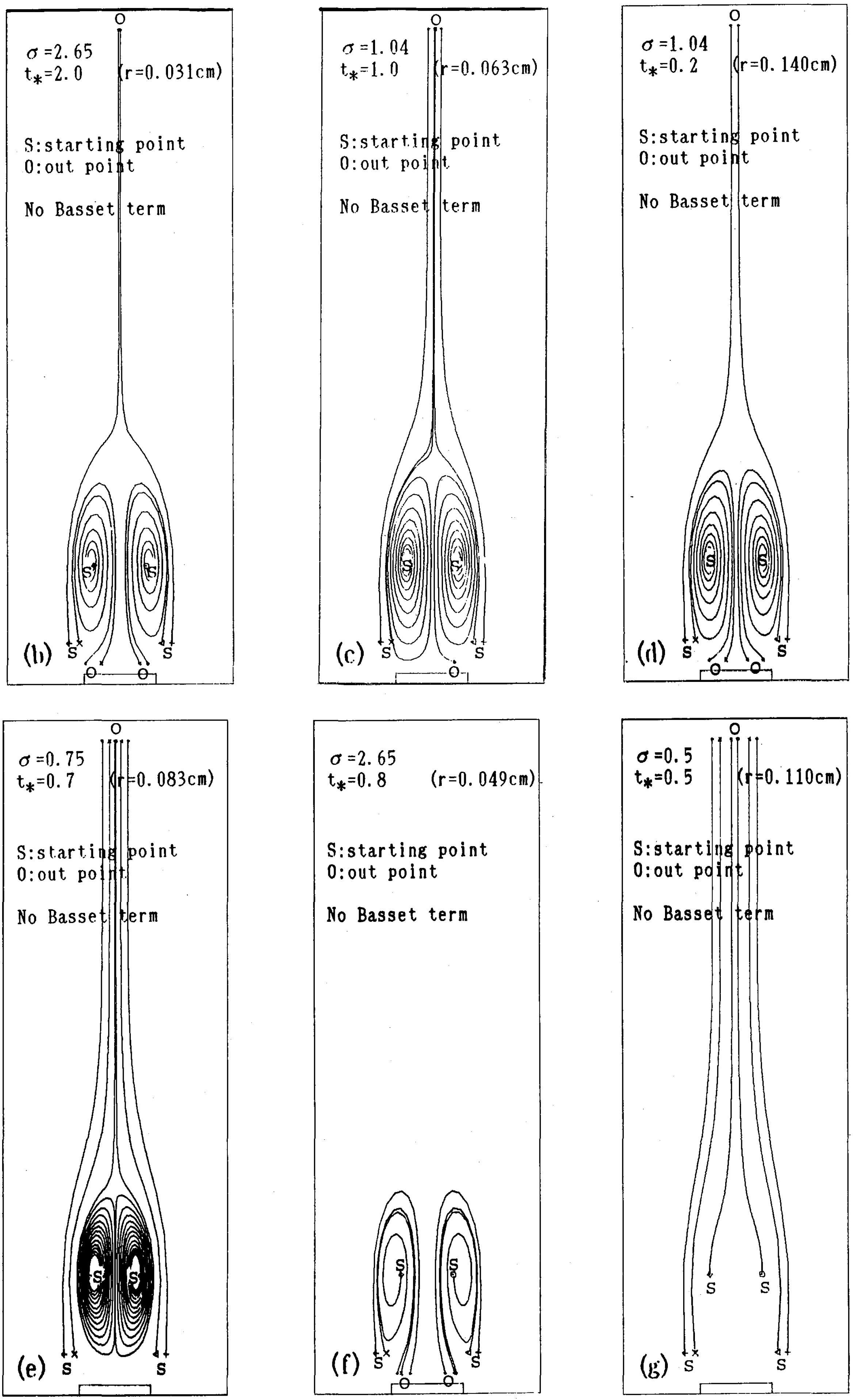


Fig. 13 Trajectory of particles (upward flow)

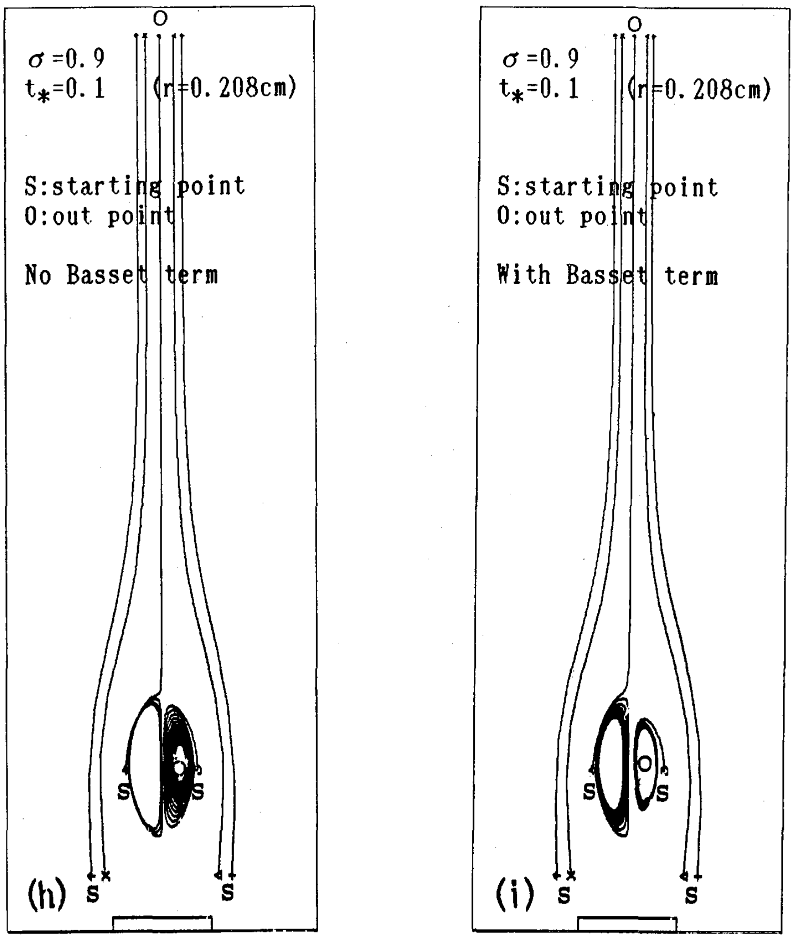


Fig. 13 Trajectory of particles (upward flow)

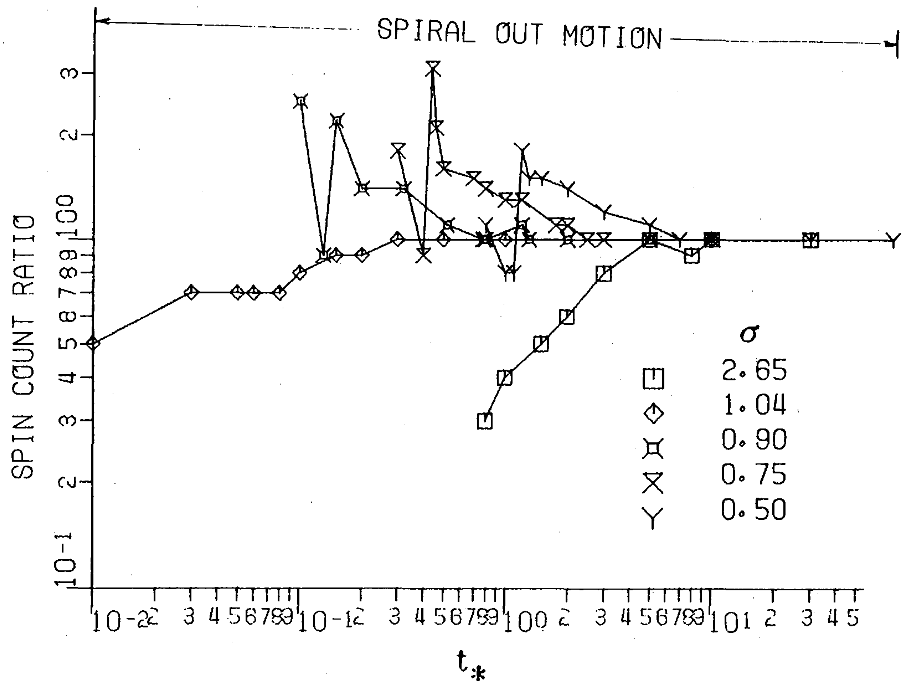


Fig. 14 Spin count of the motion of particle (upward flow)

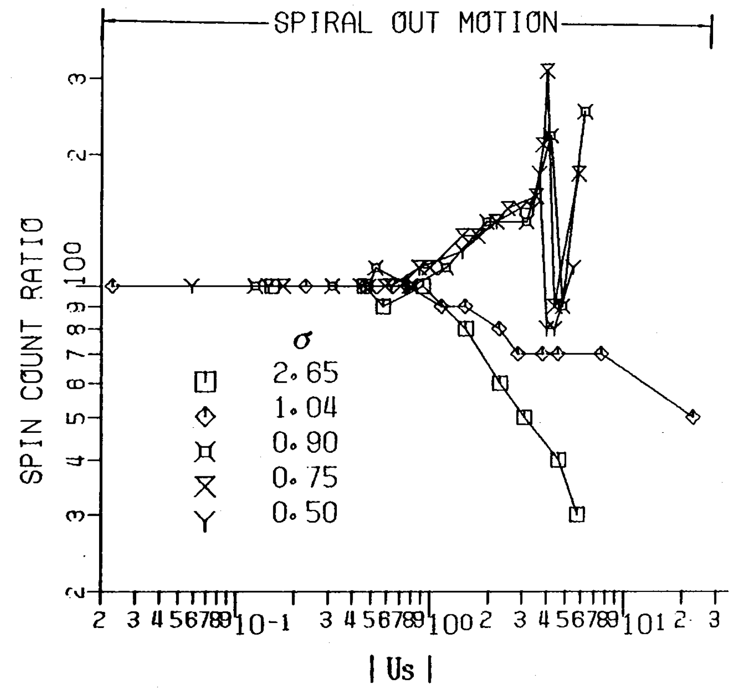


Fig. 15 Relation between $|U_S|$ and spin count ratio (upward flow)

4. Conclusion

The results in numerical experiments of the particle motion are summarized as follows.

(1) By the theoretical solution derived from the method of successive approximation, we can grasp the characteristics of the particle motion.

(2) By the presence of the Basset term, the pathlines of the spiral motion are different. Especially, it is remarkable in the case of the comparative large radius of particles.

(3) The particles in the following condition seem to be suitable for tracer particles in the flow visualization.

① $t_* > 5.0$ in the horizontal section (referring 3.(i)).

② $|U_s| < 0.05$ in the vertical section (referring 3.(ii)).

③ $|U_s| < 0.6$ in the vertical section of the upward flow (referring 3.(iii)).

(4) In the horizontal section without the gravity force, the spiral in and out motion of particles occur in the case of which the specific weight of the particle is lighter.

(5) On comparing with each pathline near the center of the spiral, the figures of pathline are different under influences of the gravity force and the size of the particle.

The problem of how to deal with the motion of particles in the unsteady flow hereafter still remains. Further a study will be needed for the motion of a group of particles.

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