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Effects of Understanding Relational Concepts between Duration, Distance and Speed on the Achievement of Math "Speed" in the 5th Grade¹

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In this paper, we tried to answer the following three questions: First, how do children acquire the qualitative relational concepts between duration, distance, and speed? Second, is there a relationship between the understanding of these relational concepts and the achievement in the math "speed" in elementary schools? Third, what would be suggested from the answers to Questions 1 and 2 in regard to the improvement in teaching of the math "speed"?

[Keywords: Relational concepts, Development, Math "speed"]

Time and space are basic frameworks of the physical world. And a movement appears, connecting one temporal and spatial point and another temporal and spatial point. Therefore, it is reasonable to say that the concepts of duration, distance, and speed are essential to a logical recognition of the physical world. On the other hand, the physical world has been recognized as being filled with many relational concepts to be discovered from ancient times. From these two points, the formula, $\text{speed} = \text{distance}/\text{duration}$, must be one of the basic relational concepts that serves as a framework of the physical world in cognition.

Then, there appear three questions. First, how do children acquire the qualitative relational concepts between duration, distance, and speed? Second, is there a relationship between the understanding of these relational concepts and the achievement in the math "speed" in elementary schools? Third, what would be suggested from the answers to Questions 1 and 2 in regard to the improvement in teaching of the math "speed"?

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As the answers to Question 1 have been already presented in Matsuda (1994, 2001), the description given in the following only refers to it briefly.

**Question 1-1: A Cross-sectional Study about the Development of Relational Concepts
between Duration, Distance, and Speed**

First, I examined the first question by performing a cross-sectional study.

Method

Participants were 222 children, with ages ranging from four years zero months to 11 years 11 months of age.

Three different values of duration, distance, and speed were used, respectively, that is: durations, which were made concrete as durations of toy-trains' whistles, were 2.0 s, 5.0 s, and 12.5 s; distances, which were made concrete as distances from a start point station to destination stations, were 20 cm, 50 cm, and 125 cm; and speeds, which were made concrete as speeds of the toy trains, were 4 cm/s, 10 cm/s, and 25 cm/s. As you can see, in every variable, the three stimuli were set at 1 : 2.5 : 2.5². The task consisted of six sessions, which were designated as $T=S\pm$, $T=D\pm$, $D=S\pm$, $D=T\pm$, $S=T\pm$, and $S=D\pm$.

The following is the procedure in the case of $T=S\pm$: First, the experimenter showed the participant that the train which traveled at 10 cm/s ran to the station 50 cm away from the start point station while whistle blew for 5.0 s; Second, the experimenter asked the participant to guess what station the fastest train would reach in the time that the same whistle blew; Third, the experimenter asked the participant reasons for his or her answer; Fourth, the experimenter let the train run according to the participant's answer to give the participant concrete feedback. The same processes were repeated by using the slowest train, though the first demonstration was omitted; Finally, the experimenter asked whether the duration had been constant or not during the three trial runs. The other five sessions were conducted in the same way, except that attributes of duration, distance, and speed were systematically interchanged with one another.

There were the following three kinds of measure, for example, in the case of Session $T=S\pm$: the distances that were chosen, quality of reasons why those distances were chosen, and the degree of recognition of the fact that the duration had been constant.

Results and Discussion

Six developmental phases were found based on those three kinds of measure.

Phase 0: A few 4- and 5-year olds only could guess the direct relation of either duration or speed to distance with some success.

Phase 1: Children displayed a considerable understanding of the direct relations between duration and distance, and distance and speed. However, they were limited in their ability to verbalize their reasoning processes. They seemed to have no knowledge of the inverse relation between duration and speed. This phase was most common among the 4-year-olds.

Phase 2: The children occasionally displayed understanding of the inverse relation between duration and speed. However, the direct relation between distance and speed was sometimes confused. They seemed to be unaware of the third dimension that was constant. About two-thirds of the 5-year-olds belonged to this phase, as did 40% of the 6- and 7-year-olds.

Phase 3: Children almost fully understood the two direct relations and the inverse relation. Their reasoning processes were also verbalized fairly well, although the third dimension still tended to be ignored. About 40% of each of the 6- to 9-year-olds belonged to this phase.

Phase 4: The third dimension began to be considered, although it was still rarely referred to spontaneously by each child. From 40 to 50% of the 8-year and older children belonged to this phase.

Phase 5: They seemed to be able to judge consciously based on the duration-distance-speed system. About 30% of the 10-year-olds and 50% of the 11-year-olds belonged to this phase.

Question 1-2: A Longitudinal Study to See Whether the Same Children Develop through These Phases

The first question was also examined by a longitudinal study, to test the validity of the emerging developmental phases.

Method

Twenty-nine children participated in the experiment once a year starting from K or the first grade to the 6th grade. There were two kinds of tasks: a simple task and a complex task. The simple task was the same one used in the cross-sectional study. The complex task, which used the same apparatus and the same values of the stimuli as the simple task did, consisted of six problems designated as T+D+, T-D-, D+S+, D-S-, S+T-, and S-T+. These problems were carried out to directly examine whether a participant made guesses based on two-by-two

relations or on the duration-distance-speed system. In the case of S + T-, for example, the participant was asked how distance would change, if the speed (S) increased (+) and duration (T) decreased (-). Then, they were given concrete feedback.

Results and discussion

Figure 1 shows the percentages of the participants belonging to each phase in each age for both cross-sectional and longitudinal studies. The figure indicates that the children who participated in the longitudinal study showed accelerated development after 9 years old, when compared to the children in the cross-sectional study. They seemed to be able to transfer more easily to Phase 5. This finding shows that the repeated experiences with appropriate feedback were very effective, even if only given once a year.

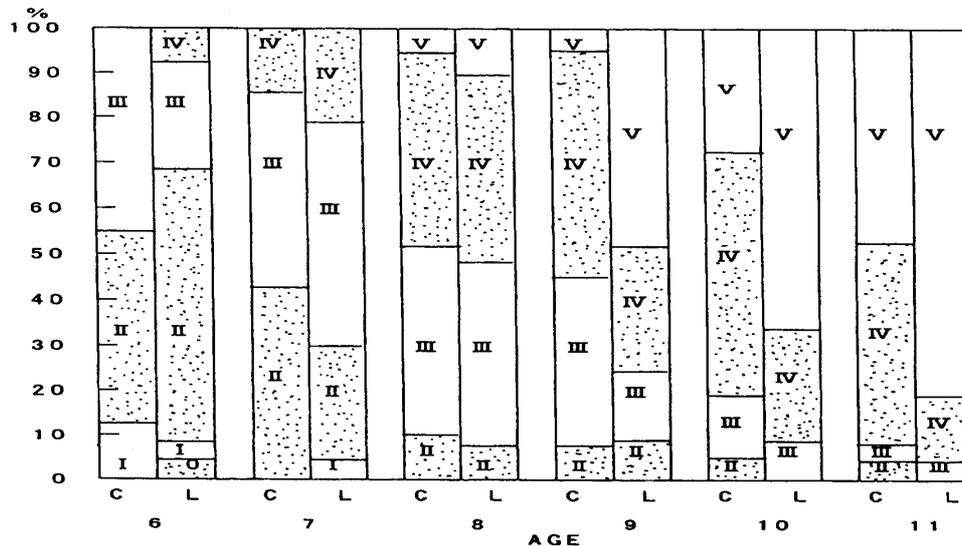


Figure 1. Percentages of participants belonging to the developmental phases for each age. C: Cross-sectional study; L: Longitudinal study.

Question 2: Relationships between Understandings of These Relational Concepts and Achievement in Math “Speed” of Elementary School Children

The achievement of math “speed” of the participants in the longitudinal study in the 5th grade was examined, because they showed the better understanding of the relational concepts between duration, distance, and speed than usual. Would they show better achievements than students who had not participated to the longitudinal study ?

Method

Participants in the experimental group ($n = 27$) were the children in the longitudinal study. There were 29 children who participated in the longitudinal study, but two of them learned math in the special class because of their a little difficulty of learning so that they were excluded from the experimental group. The participants of the control group ($n = 75$) were children in the 5th grade who did not participate in this study, though attending to the same school.

We used two kinds of tests, Test 1 and Test 2. Test 1 was a typical achievement test to see effects of the math "speed" teaching, consisted of three types of problems: Type 1 consisted of problems that required calculation of duration, distance, or speed by simply applying the formula, $\text{speed} = \text{distance}/\text{duration}$. The answers required students to use appropriate units such as, s, cm, cm/s, and so on; Type 2 problems included exchanging the units of speed such as cm/s, cm/min, and km/hr, etc. for each other, in order to compare speeds of two moving objects; Type 3 problems did not directly related with the formula, $\text{speed} = \text{distance}/\text{duration}$. For example, speed of typing. Test 2 consisted of the word problems to see the understanding of the relational concepts between duration, distance and speed, such as, "When speed becomes three times faster, distance becomes () times longer, if the running duration is the same". These tests were carried out before and after the math "speed" teaching as shown in Figure 2.

In the math "speed" teaching, teachers did not emphasize the relationship between duration, distance, and speed, but emphasized the definition of speed, that is, speed is distance per a unit duration and calculation by using the formula, $\text{speed} = \text{distance}/\text{duration}$. This teaching way is formal in Japan and, it is well-known that math "speed" is one of the most difficult contents in math for elementary school children. It took about eight hours.

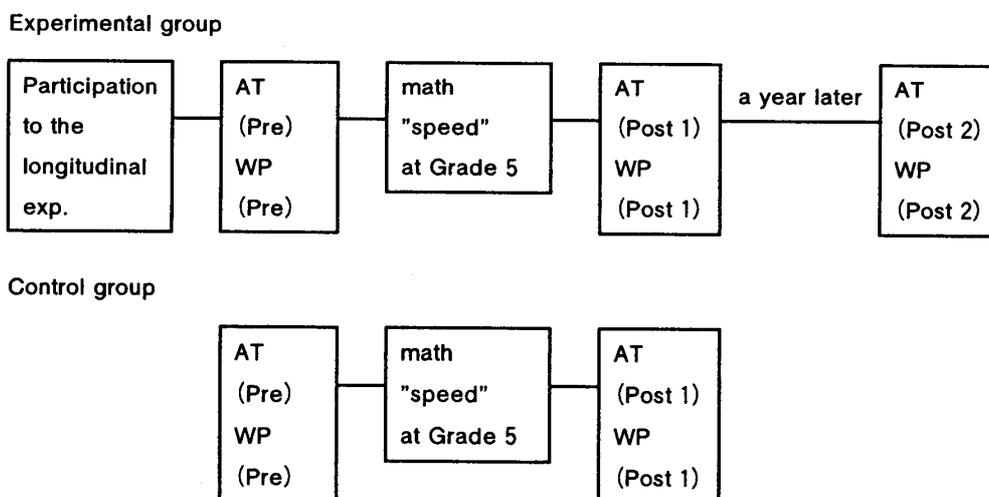


Figure 2. The procedure. AT: Achievement test; WP: Word problems.

Results and discussion

In the experimental group, the fact that the participants had been in Phase 5 just before given the math "speed" teaching, was considerably important for the participants to get over .50 in the rate of correct responses in the achievement test, as you can see in Figure 3.

Table 1 shows rates of correct responses in the achievement test that carried out before and after the math "speed" teaching. This table indicates that the experimental group obtained significantly better achievement test scores, especially in Type 2 problems after the teaching, while there were no differences between the two groups before the teaching. There seem to be two reasons for this better achievement in the experimental group. First, the children in the experimental group had generally acquired better relational concepts between duration, distance, and speed, so that they could use more resources for the complex manipulation of units. Second, they had repeatedly observed the uniform linear movement of the toy trains in the longitudinal experiment, so that it might have been easier for them to imagine the situation of the question, for example, "Suppose that this train runs 20 cm for one second. How long would it run, if it would continue to run at the same speed for an hour?"

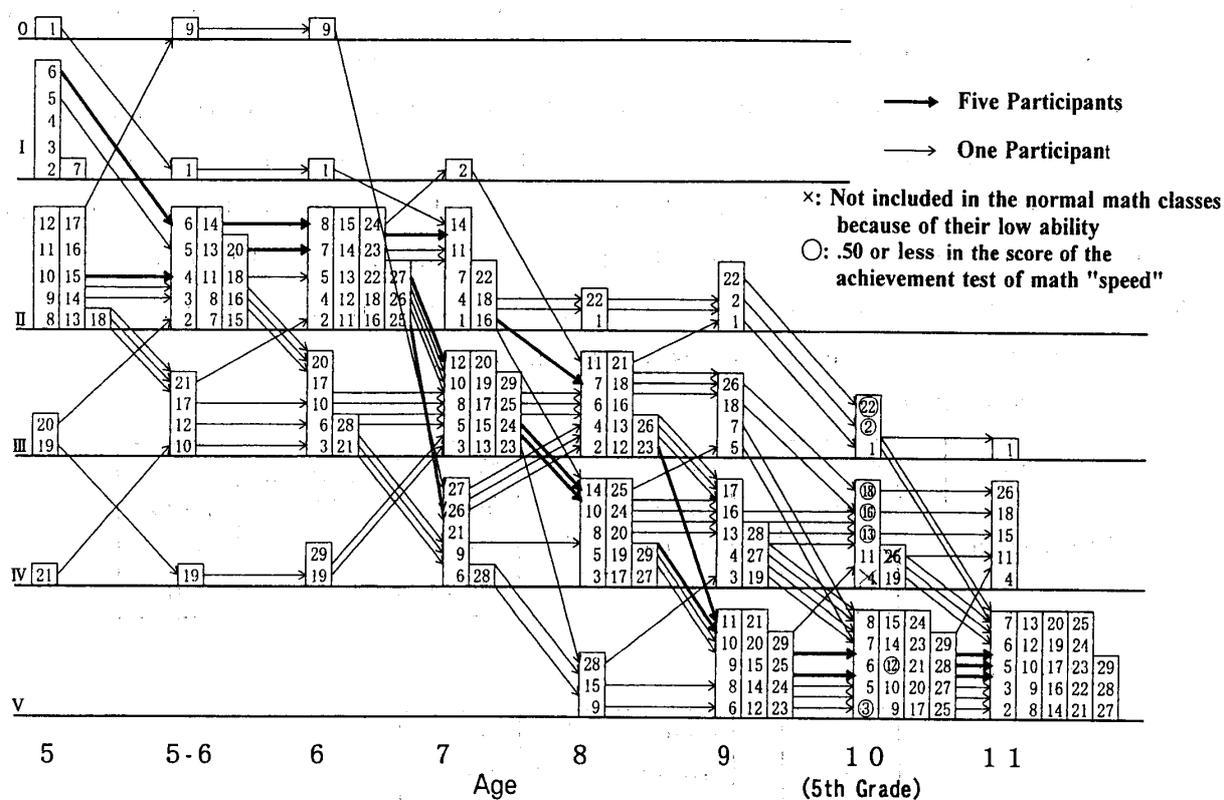


Figure 3. Phases of participants at each age and their transitions from one phase to another in the longitudinal study (numbers mean participants' ID) and their achievement test scores just after the math "speed" at the 5th grade.

Table 1 Rates of correct responses in the pre- and post-achievement tests

Group	n	Pre	Post			
		Total	Total	Type 1	Type 2	Type 3
Experimental	27	0.35	0.67	0.77	0.63	0.60
Control	75	0.33	0.55	0.66	0.33	0.52

Table 2 shows rates of correct responses in the word problems dividing the participants into two groups, that is, the upper group with good achievement test scores and the lower group with bad achievement test scores. In the pre-word problems, there were no significant differences between the upper and lower groups both in the experimental and control groups, though the experimental group was significantly better than the control group as a whole. After math "speed" teaching, the scores of the upper group were significantly better than the lower group. The lower group did not catch up at all even after a year later. On the contrary, the differences became much larger. Therefore, the construction of the qualitative relational concepts between duration, distance, and speed and the understanding of the math "speed" seem to be interdependent. These findings suggest that the present way to teach the math "speed" that ignores the informal prior knowledge about these relational concepts is not effective.

Table 2 Rates of correct responses in pre- and post-word problems

Group	n	Pre	Post 1	Post 2
Experimental				
Upper	19	0.58	0.60	0.71
Lower	7	0.61	0.48	0.39
Control				
Upper	35	0.40	0.51	—
Lower	40	0.34	0.35	—

Note. Upper: Rates of correct answers in the achievement test were over .50; Lower: Those were .50 or less.

These findings strongly suggest the following: The children with a high level of understanding the qualitative relational concepts before the math "speed" teaching could easily understand the math "speed" because they could use these relational concepts as a scaffold, even if the math "speed" teaching was not intentionally based on these relational concepts. The math "speed" teaching strengthened these relational concepts a little further. On the contrary, the children

with a low level of understanding the qualitative relational concepts before the math “speed” teaching couldn’t easily understand because it was learned without relation to their prior knowledge of the relational concepts. As the result, their prior informal knowledge of the relational concepts couldn’t be developed but rather was destroyed by the math “speed” teaching.

Question 3: How to Improve Math “Speed” Teaching

The last question is how to improve the teaching of math “speed.” The findings mentioned above strongly suggest the following about this problem:

First, concerning the matter before entering the usual math “speed” teaching, teachers should elaborate on the relational concepts between duration, distance, and speed to the level of Phase 5.

Second, in the math “speed” teaching, it would be important to use prior knowledge effectively. That is, teachers should introduce the formula, speed equals distance divided by duration, as the representation of the qualitative relationship between the three concepts and after that, lead to the idea that speed is distance per unit duration. Another suggestion is to use explicitly a linear uniform movement. In that case, it is expected for children to understand intuitively the interchangeability of various speed units.

References

- Matsuda, F. (1994) . Concepts about interrelations among duration, distance, and speed in young children. *International Journal of Behavioural Development*, 17, 553-576.
- Matsuda, F. (2001) . Development of concepts of interrelationships among duration, distance, and speed. *International Journal of Behavioural Development*, 25, 466-480.